Fall 2016. Math 5615H.
Homework 4 (due on Wednesday, October 19).

#1. Let $f$ be a mapping of $A$ to $B$. Show that for each $B_1 \subseteq B$ and $B_2 \subseteq B$, their inverse images satisfy the properties

(i) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$,  
(ii) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

#2. Let $f$ be a mapping of $A$ to $B$. Verify whether or not the images of subsets $A_1 \subseteq A$ and $A_2 \subseteq A$ in general satisfy the properties

(i) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$,  
(ii) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$.

#3. The set difference of $A$ and $B$ by definition is

$$A \setminus B := \{ x \in A : x \notin B \}.$$ 

Simplify the expressions

(a) $A \setminus (B \setminus A)$,  
(b) $A \setminus (A \setminus B)$,  
(c) $A \cap (B \setminus A)$.

#4. The symmetric difference of $A$ and $B$ by definition is

$$A \Delta B := (A \setminus B) \cup (B \setminus A).$$ 

Show that for an arbitrary set $C$,

$$A \Delta B \subseteq (A \Delta C) \cup (B \Delta C).$$

#5. Show that for an arbitrary sequence $E_1, E_2, \ldots$ of sets, the set

$$\bigcup_{n=1}^{\infty} \left( \bigcap_{k=n}^{\infty} E_k \right)$$

is contained in

$$\bigcap_{n=1}^{\infty} \left( \bigcup_{k=n}^{\infty} E_k \right).$$