#1. Let \( A > 0 \), and
\[
x_1 = a > 0, \quad x_{n+1} = \frac{1}{2} \cdot \left( x_n + \frac{A}{x_n} \right) \quad \text{for} \quad n = 1, 2, \ldots
\]
Show that the sequence \( \{x_n\} \) converges and find its limit.

#2. Show that a functions \( y = f(x) \) is continuous on \([0, 1]\) if and only if its graph
\[
\Gamma := \{(x, y) \in \mathbb{R}^2 : \ x \in [0, 1], \ y = f(x)\}
\]
is a compact in \( \mathbb{R}^2 \). Use without proof the fact that \( y = f(x) \) is continuous on \([0, 1]\) if and only if from \( x_n \in [0, 1] \) and \( x_n \to a \in [0, 1] \) as \( n \to \infty \), it follows that \( f(x_n) \to f(a) \) as \( n \to \infty \).

#3. Let \( f(x, y) \) be a continuous function for \( 0 \leq x \leq T = \text{const} < \infty, \ y \in \mathbb{R}^1 \), satisfying the Lipschitz condition with respect to \( y \) with a constant \( K > 0 \):
\[
|f(x, y_1) - f(x, y_2)| \leq K \cdot |y_1 - y_2|, \quad \forall x \in [0, T]; \ y_1, y_2 \in \mathbb{R}^1.
\]
Let \( X \) be a set of all continuous functions on \([0, T]\). Define a mapping \( T \) from \( X \) to \( X \) as follows: for any continuous function \( y = y(x) \) on \([0, T] \), \( T(y) \) is a continuous function on \([0, T]\) with values
\[
(T(y))(x) := y_0 + \int_0^x f(t, y(t)) \, dt, \quad \text{where} \quad y_0 = \text{const} \in \mathbb{R}^1.
\]
Introduce the distance between \( y_1 \) and \( y_2 \) in \( X \) as
\[
d(y_1, y_2) := \int_0^T e^{-2Kx} |y_1(x) - y_2(x)| \, dx.
\]
Show that
\[
d(T(y_1), T(y_2)) \leq \frac{1}{2} \cdot d(y_1, y_2), \quad \forall y_1, y_2 \in X.
\]