Math 5616H, Sec. 1. Information on Final Exam, Tuesday, May 9, 2017, VinH 207, 10:30 am – 12:30 pm.

There will be 6 problems, some with subproblems. No books, no notes. Calculators are permitted, but for full credit, you have to show step by step calculations. Any additional reference material will be included into the booklet with problems.

The exam will be based on the following material in the textbook.
Chapter 5. Differentiation.
Chapter 7. Sequences and Series of Functions. No Stone-Weierstrass theorem and related material (algebras, etc).
Chapter 8. Some special functions. You can use any definition and properties of elementary functions $e^x$, $\cos x$, $\sin x$, $\ln x$.

The exam will include the following two take-home problems.

**Problem 1.** Consider Legendre polynomials

$$L_n(x) := c_n \cdot \frac{d^n}{dx^n}[(x^2 - 1)^n], \quad \text{where} \quad c_n := \frac{1}{2^n \cdot n!}, \quad n = 0, 1, 2, \ldots$$

(a). Show that

$$\int_{-1}^{1} L_m L_n \, dx = 0 \quad \text{if} \quad m \neq n.$$

(b). Prove the identities

$$(n + 1)L_{n+1}(x) = (2n + 1)x \, L_n(x) - n \, L_{n-1}(x) \quad \text{for} \quad n = 1, 2, \ldots$$

**Problem 2.** Find the minimum of

$$f(c_0, c_1, c_2) := \int_{-1}^{1} \left(x^3 - c_0 - c_1 x - c_2 x^2\right)^2 \, dx \quad \text{over} \quad c_0, c_1, c_2 \in \mathbb{R}.$$

For solving these problems, and also the remaining 4 problems, you can use any method, not necessarily based on the material in this course. It is recommended to review solutions of HW and MT problems, with special attention to the following facts and techniques.

1. Taylor’s expansion of $e^x$, $\sin x$, $\cos x$, $\ln(1 + x)$, $(1 + x)^a$.
2. L’Hospital’s rule.
3. Differentiation and integration of series and integrals with respect to a parameter.

For example, formula (98) and Example 9.43 in Chapter 9 (which was not covered in details), can be considered as a special case of uniform limits. If $f$ and $f'_x$ are continuous in $t, x$, then

$$\frac{d}{dx} \int_{a}^{b} f(t, x) \, dt = \int_{a}^{b} f'_x(t, x) \, dt.$$