Homework #3 (due on October 17): Sec.2.3: #6; Sec.2.5: #2,4; Sec.3.1: #2,10; Sec.3.2: #4,10; Sec.3.3: #6.

Sec.2.3: #6. Suppose that when a machine is adjusted properly, 50 percent of the items produced by it are of high quality and the other 50 percent are of medium quality. Suppose, however, that the machine is improperly adjusted during 10 percent of the time and that, under these conditions, 25 percent of the items produced by it are of high quality and the 75 percent are of medium quality.

a. Suppose that 5 items produced by the machine at a certain time are selected at random and inspected. If 4 of these items are of high quality and 1 item is of medium quality, what is the probability that the machine was adjusted properly at that time?

b. Suppose that 1 additional item, which was produced by the machine at the same time as the other 5 items, is selected and found to be of medium quality. What is the new posterior probability that the machine was adjusted properly?

Sec.2.5: #2. Suppose that a fair coin is tossed repeatedly and independently until both a head and a tail have appeared at least once. (a) Describe the sample space of this experiment. (b) What is the probability that exactly three tosses will be required?

Sec.2.5: #4. Suppose that $A$ and $B$ are independent events such that $P(A) = 1/3$ and $P(B) > 0$. What is the value of $P(A \cup B^c \mid B)$?

Sec.3.1: #2. Suppose that a random variable $X$ has a discrete distribution with the following p.f.

$$f(x) = \begin{cases} cx & \text{for } x = 1, \ldots, 5, \\ 0 & \text{otherwise}. \end{cases}$$

Determine the value of the constant $c$. 

(over)
**Sec.3.1: #10.** A civil engineer is studying a left-turn lane that is long enough to hold 7 cars. Let $X$ be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(8 - x)$ for $x = 0, \ldots, 7$ (the possible values of $X$).

a. Find the p.f. of $X$.
b. Find the probability that $X$ will be at least 5.

**Sec.3.2: #4.** Suppose that the p.d.f. of a random variable $X$ is as follows:

$$f(x) = \begin{cases} cx^2 & \text{for } 1 \leq x \leq 2, \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of the constant $c$ and sketch the p.d.f.
b. Find the value of $P(X > 3/2)$.

**Sec.3.2: #10.** Suppose that the p.d.f. of a random variable $X$ is as follows:

$$f(x) = \begin{cases} \frac{c}{(1-x)^{1/2}} & \text{for } 0 < x < 1, \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of the constant $c$ and sketch the p.d.f.
b. Find the value of $P(X \leq 1/2)$.

**Sec.3.3: #6.** Suppose that the d.f. of a random variable $X$ is as follows:

$$F(x) = \begin{cases} e^{x-3} & \text{for } x \leq 3, \\ 1 & \text{for } x > 3. \end{cases}$$

Find and sketch the p.d.f. of $X$. 