Homework #4 (due on October 31):
Sec. 3.4: #4; Sec. 3.5: #2, 10; Sec. 3.6: #6, 10; Sec. 3.7: #2, 4.

Sec. 3.4: #4. Suppose that $X$ and $Y$ have a continuous joint distribution for which the joint p.d.f. is defined as follows:

$$f(x, y) = \begin{cases} cy^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine (a) the value of the constant $c$; (b) $P(X + Y > 2)$; (c) $P(Y < 1/2)$; (d) $P(X \leq 1)$; (e) $P(X = 3Y)$.

Sec. 3.5: #2. Suppose that $X$ and $Y$ have a discrete joint distribution for which the joint p.f. is defined as follows:

$$f(x, y) = \begin{cases} \frac{1}{30}(x + y) & \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the marginal p.f.’s of $X$ and $Y$.
(b) Are $X$ and $Y$ independent?

Sec. 3.5: #10. Suppose that a point $(X, Y)$ is chosen at random from the circle $S$ defined as follows:

$$S = \{(x, y) : x^2 + y^2 \leq 1\}.$$

(a) Determine the joint p.d.f. of $X$ and $Y$, the marginal p.d.f. of $X$, and the marginal p.d.f. of $Y$.
(b) Are $X$ and $Y$ independent?

Sec. 3.6: #6. Suppose that the joint p.d.f. of two random variables $X$ and $Y$ is as follows:

$$f(x, y) = \begin{cases} c \sin x & \text{for } 0 \leq x \leq \pi/2 \text{ and } 0 \leq y \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

Determine (a) the conditional p.d.f. of $Y$ for every given value of $X$, and (b) $P(1 < Y < 2 \mid X = 0.73)$. 

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**Sec. 3.6:** #10. In a large collection of coins, the probability $X$ that a head will be obtained when a coin is tossed varies from one coin to another and the distribution of $X$ in the collection is specified by the following p.d.f.:

$$f_1(x) = \begin{cases} 
6x(1-x) & \text{for } 0 < x < 1, \\
0 & \text{otherwise.}
\end{cases}$$

Suppose that a coin is selected at random from the collection and tossed once, and that a head is obtained. Determine the conditional p.d.f. of $X$ for this coin.

**Sec. 3.7:** #2. Suppose that three random variables $X_1$, $X_2$, and $X_3$ have a mixed joint distribution with p.f./p.d.f.

$$f(x_1, x_2, x_3) = \begin{cases} 
cx_1^{1+x_2+x_3}(1-x_1)^{3-x_2-x_3} & \text{if } 0 < x_1 < 1 \\
0 & \text{and } x_2, x_3 \in \{0, 1\}, \\
& \text{otherwise.}
\end{cases}$$

(Notice that $X_1$ has a continuous distribution and $X_2$ and $X_3$ have discrete distributions.) Determine (a) the value of the constant $c$; (b) the marginal joint p.f. of $X_2$ and $X_3$; (c) the conditional p.d.f. of $X_1$ given $X_2 = 1$ and $X_3 = 1$.

**Sec. 3.7:** #4. Suppose that a point $(X_1, X_2, X_3)$ is chosen at random, that is, in accordance with a uniform p.d.f., from the following set $S$:

$$S = \{(x_1, x_2, x_3) : 0 \leq x_i \leq 1 \text{ for } i = 1, 2, 3\}.$$ Determine

a. $P\left[\left(X_1 - \frac{1}{2}\right)^2 + \left(X_2 - \frac{1}{2}\right)^2 + \left(X_3 - \frac{1}{2}\right)^2 \leq \frac{1}{4}\right]$,  

b. $P(X_1^2 + X_2^2 + X_3^2 \leq 1)$. 

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