Math 5651: Basic Theory of Probability and Statistics: Fall 2019

Homework #5 (due on Tuesday, November 19):
Sec. 3.8: #2, 8; Sec. 3.9: #4, 6; Sec. 3.11: #20;
Sec. 4.1: #8; Sec. 4.2: #2, 8, 10.

Sec. 3.8: #2. Suppose that a random variable $X$ can have each of the seven values $-3, -2, -1, 0, 1, 2, 3$ with equal probability. Determine the p.f. of $Y = X^2 - X$.

Sec. 3.8: #8. Suppose that the p.d.f. of $X$ is as follows:

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0, \\ 0 & \text{for } x \leq 0. \end{cases}$$

Determine the p.d.f. of $Y = X^{1/2}$.

Sec. 3.9: #4. Suppose that $X_1$ and $X_2$ have a continuous joint distribution for which the joint p.d.f. is as follows:

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{for } 0 < x_1 < 1 \text{ and } 0 < x_2 < 1, \\ 0 & \text{otherwise}. \end{cases}$$

Find the p.d.f. of $Y = X_1X_2$.

Sec. 3.9: #6. Let $X$ and $Y$ be random variables for which the joint p.d.f. is as follows:

$$f(x, y) = \begin{cases} 2(x + y) & \text{for } 0 \leq x \leq y \leq 1, \\ 0 & \text{otherwise}. \end{cases}$$

Find the p.d.f. of $Z = X + Y$.

Sec. 3.11: #20. Suppose that the random variables $X, Y,$ and $Z$ have the following p.d.f.:

$$f(x, y, z) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \text{ and } 0 < z < 1, \\ 0 & \text{otherwise}. \end{cases}$$

Evaluate $P(3X > Y \mid 1 < 4Z < 2)$. (over)
Sec.4.1: #8. Suppose that $X$ and $Y$ have a continuous joint distribution for which the joint p.d.f. is as follows:

$$f(x, y) = \begin{cases} 12y^2 & \text{for } 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise}. \end{cases}$$

Find the value of $E(XY)$.

Sec.4.2: #2. Suppose that three random variables $X_1, X_2, X_3$ form a random sample from a distribution for which the mean is 5. Determine the value of

$$E(2X_1 - 3X_2 + X_3 - 4).$$

Sec.4.2: #8. Suppose that a class contains 10 boys and 15 girls, and suppose that 8 students are to be selected at random from the class without replacement. Let $X$ denote the number of boys that are selected, and let $Y$ denote the number of girls that are selected. Find $E(X - Y)$.

Sec.4.2: #10. Suppose that a fair coin is tossed repeatedly until a head is obtained for the first time. (a) What is the expected number of tosses that will be required? (b) What is the expected number of tails that will be obtained before the first head is obtained?