#1. Let $F$ be a compact subset of $\mathbb{R}^n$. Show that there are points $x_0, y_0 \in F$, such that
\[
\text{diam} F := \sup \{|x - y| : x, y \in F\} = |x_0 - y_0|.
\]

*Hint.* There are sequences $\{x_j\}$, $\{y_j\}$ in $F$, such that
\[
\text{diam} F = \lim_{j \to \infty} |x_j - y_j|.
\]

#2. Let $f(x)$ be a real function on a compact set $E \subset \mathbb{R}^1$. Show that $f$ is continuous on $E$ if and only if its graph
\[
\Gamma = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \in E, x_2 = f(x_1)\}
\]
is a compact set in $\mathbb{R}^2$.

*Hint.* You can use the fact that $f(x)$ is continuous on $E$ $\iff$
From $\{x_0, x_1, x_2, \ldots\} \subset E$ and $x_0 = \lim x_j$ it follows $f(x_0) = \lim f(x_j)$.

#3. Let a subset $E \subset \mathbb{R}^n$ be nonempty, open and closed simultaneously. Show that $E = \mathbb{R}^n$.

*A possible way.* Suppose that both $E$ and $E^c$ are nonempty. Then there are points $x_0 \in E$ and $y_0 \in E^c$. They are connected by the segment
\[
[x_0, y_0] := \{x(t) := x_0 + t(y_0 - x_0) : 0 \leq t \leq 1\} \text{ in } \mathbb{R}^n.
\]
Finally, consider the point
\[
z_0 := x(t_0), \text{ where } t_0 := \sup \{t \in [0, 1] : x(t) \in E\}.
\]

In the remaining two problems, the following notations are used:
\[
A^c := \mathbb{R}^n \setminus A, \quad ^\circ A - \text{the interior of } A, \quad \overline{A} - \text{the closure of } A.
\]

#4. For an arbitrary set $E \subset \mathbb{R}^n$, show that
\[
\left( ^\circ E \right)^c = \overline{E}^c.
\]

#5. Denote $^\circ E$ - the interior of $E$, $\overline{E}$ - the closure of $E$, $\overline{\overline{E}}$ - the interior of $\overline{E}$, etc. Give examples of sets $E$ such that:
\[(a) \ ^\circ E \neq E, \quad (b) \ \overline{\overline{E}} \neq \overline{E}.
\]