#1. Let $K$ be a family of all closed subset of $[0,1] \times [0,1]$ with respect to the Euclidean distance. Show that $K$ is a metric space with the Hausdorff distance

$$
\rho(A, B) := \max \left\{ \max_{x \in A} \text{dist}(x, B), \max_{y \in B} \text{dist}(y, A) \right\}, \\
\text{dist}(x, B) := \min_{y \in B} |x - y|, \quad \text{etc.}
$$

#2. Show that in the previous problem, the metric space $(K, \rho)$ is compact.

#3 (Problem 63, p. 138). Let $K(x, y)$ be a continuous function on $[0,1] \times [0,1]$. Consider the metric space $(C([0,1]), \rho)$, where

$$
\rho(f, g) := \max_{[0,1]} |f - g|.
$$

Show that the family of functions

$$
A := \left\{ F(x) := \int_0^1 K(x, y) f(y) \, dy : \ f \in C([0,1]), \ \max_{[0,1]} |f| \leq 1 \right\}
$$

is a precompact subset of $(C([0,1]), \rho)$. Verify whether or not it is compact.

#4 (§4.3, p. 147.) Let $(\mathcal{F}, \subseteq)$ be a filter directed under reverse inclusion, i.e.

$$
F_1 \subseteq F_2 \iff F_2 \subseteq F_1.
$$

A net $< x_F >_{F \in \mathcal{F}}$ is associated to $\mathcal{F}$ if $x_F \in F$ for every $F \in \mathcal{F}$. Show that

$$
\mathcal{F} \rightarrow x \iff \text{every associated net } < x_F >_{F \in \mathcal{F}} \rightarrow x.
$$