1. Write the first four terms of the Taylor series for the function

\[ f(x, y) = \frac{1}{1 - x - y^2} \]

around the point \((x, y) = (0, 0)\)

Answer: \(1 + x + (y^2 + x^2) + (2xy^2 + x^3)\).

2. Write the first four terms of the Taylor series for the function

\[ f(x, y) = \frac{x}{1 - x^2 - y^3} \]

around the point \((x, y) = (0, 0)\)

Answer: \(f(x, y) = x + x^2 + xy^3 + x^5 + \cdots\).

3. Find an approximate value of the expression \(\sin(42^\circ)\) using the linear approximation.

Answer: \(\sin(42^\circ) \approx 0.67\).

4. Find the linear and quadratic approximation for the expression

\[ \tan\left(\frac{\pi + 0.01}{3.97}\right) \]

using the first and second order approximations. Compare your answer to the exact values.

Answer: linear approximation \(\approx 1.01678\)
Answer: quadratic approximation \(\approx 1.01704\)
Answer: the exact value is 1.01705

5. Determine the second-order Taylor formula for the function \(f(x, y) = e^{x+y}\) around the point \(x_0 = 0, y_0 = 0\)

Answer: \(f(x, y) = 1 + x + y + \frac{1}{2}x^2 + xy + \frac{1}{2}y^2 + R_2\).

6. Determine the second-order Taylor formula for the function

\[ f(x, y) = e^{(x-1)^2}\cos y \]

expanded about the point \(x_0 = 1, y_0 = 0\)

Answer: \(f(h_1, h_2) = 1 + (h_1)^2 + \frac{1}{2}(h_2)^2 + R_2 \left((1, 0), (h_1, h_2)\right)\)

where \(\frac{R_2 \left((1, 0), (h_1, h_2)\right)}{\| (h_1, h_2) \|} \to 0\) as \(\| (h_1, h_2) \| \to 0\).