11.8 Go over quizzes!

84.5]

Factor Theorem → \( x - c \) is a factor of \( f(x) \) iff \( f(c) = 0 \)

# of Real Zeros \( \rightarrow \) Polynomial can have at most \( d \) real zeros where \( d \) is the degree of the polynomial.

Rational Roots Thm. → Poly. of at least \( d \geq 1 \) with integer coefficients, \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 \), \( a_n \neq 0 \). If \( \frac{p}{q} \) is a rational root of \( f(x) \), then \( p \) is a factor of \( a_0 \) and \( q \) is a factor of \( a_n \).

Steps:

1. List all factors of \( a_0 \) → possible \( p \)’s
2. List all factors of \( a_n \) → possible \( q \)’s
3. List all \( \frac{p}{q} \) combinations.
4. Find one and use synthetic division to simplify.

**Synthetic Division**

Be sure every \( x \)’s value is accounted for!

\[
\begin{array}{c|ccc}
\text{root} & \text{coefficients of poly} \\
\hline
& a_n & a_{n-1} & \ldots & a_0 \\
\end{array}
\]

\[
\text{coefficients of new poly of deg. } d-1
\]

\[
\begin{array}{c|c|c}
\text{root} & \text{coefficients of poly} & \text{rem.} \\
\hline
& & -f(0) \text{ if } 0, \text{ then root is root } \Rightarrow \text{ rem.} \\
\end{array}
\]

**Ex.** \( x^3 + 2x^2 - 3x \) divided by \( x - 1 \)

\[
\begin{array}{c|ccc}
1 & 1 & 2 & -3 & 0 \\
\hline
-1 & 1 & 3 & 0 \\
\hline
1 & 3 & 0 & 10 \\
\end{array}
\]

\( x^2 + 3x \) easily factors into \( x(x+3) \)

Thus, \( x^3 + 2x^2 - 3x = x(x-1)(x+3) \).
ex. $x^4 + 6x - 4 \Rightarrow$ is neg. 2 a root?

\[
\begin{array}{c|cccc}
-2 & 1 & 0 & 0 & -4 \\
 \hline
 & -2 & 4 & -8 & 4 \\
\end{array}
\]

-2 is a root; $(x+2)$ is a factor

$x^3 - 2x^2 + 4x - 2$ is left

ex. of RRT/ Factor. (vs. Finding Zeros)

\[f(x) = 3x^3 + 16x^2 - 15x - 30\]

\[3(x^3 + 2x^2 - 5x - 10)\] (Factor by grouping?)

\[p: \pm 1, \pm 2, \pm 5, \pm 10\]

\[q: \pm 1\]

\[\pm 1, \pm 2, \pm 5, \pm 10 \Rightarrow \text{possible roots}\]

\[f(1) = 3(1+2-5-10) = 3(-12) = -36 \neq 0\]

\[f(-1) = 3(-1+2+5-10) = 3(-4) = -12 \neq 0\]

\[f(2) = 3(8+8-10-10) = 3(0) = 0 \checkmark -2 \text{ is a root}\]

\[
\begin{array}{c|cc}
-2 & 1 & 2 & -5 & -10 \\
 \hline
 & -2 & 0 & 10 & \\
\end{array}
\]

\[x^2 - 5 \Rightarrow x^2 - 5 = 0\]

\[x^2 = 5\]

\[x = \pm \sqrt{5}\]

Thus, [f(x) = 3(x+2)(x+\sqrt{5})(x-\sqrt{5})]

A poly. of odd degree has @ least one real zero.

[A.7]

\[i = \sqrt{-1}, i^2 = -1, i^3 = -i = -\sqrt{-1}, i^4 = 1 \Rightarrow \text{cyclic!}\]

Complex numbers = $a + bi =$ "standard form" of

\[\begin{align*}
\text{real} & \quad \text{imaginary} \\
(a+bi) + (c+di) & = (a+c) + (b+d)i \\
(a+bi) - (c+di) & = (a-c) + (b-d)i \\
(a+bi) \cdot (c+di) & = ac + cbi + adi + bdi^2 \\
& = (ac-bd) + (ad+cb)i
\end{align*}\]
\[
\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-da}{c^2+d^2}i
\]

ex.) \( z = 2 - 3i, w = 3 + 7i \)

\( z + w = (2 - 3i) + (3 + 7i) = 5 + 4i \)

\( zw = \)

\( (2-3i)(3+7i) = 6 - 9i + 14i + 21 = 27 + 5i \)

\( \frac{z}{w} \)

\( \frac{2-3i}{3+7i} = \frac{-9i-14i+21}{3+7i} = \frac{-15+23i}{3+7i} \)

\( \frac{z}{w} \)

\( \frac{2-3i}{3-7i} = \frac{-9i-14i+21}{3+7i} = \frac{-15+23i}{3+7i} \)

conjugate: \( z = a+bi \rightarrow a-bi \)

\( \overline{z} = (a+bi)(a-bi) = a^2 - b^2i^2 = a^2 + b^2 \)

Discriminant \((b^2 - 4ac)\) of Quad. Eq.

1. If \( b^2 > 4ac \), will have 2 distinct real sols.
2. If \( b^2 = 4ac \), will have 1 repeated real sol. (same root twice)
3. If \( b^2 < 4ac \), will have 2 complex conjugate sols.

*complex roots always come in pairs! (with conjugate)

ex.) Solve. \( 2x^2 + 3x = -4 \)

\[ 2x^2 + 3x + 4 = 0 \]

\[ x = \frac{-3 \mp \sqrt{9 - 32}}{4} = \frac{-3 \mp \sqrt{-23}}{4} = \frac{-3 \pm i\sqrt{23}}{4} \]

ex.) If \( 2 + 3i \) is a sol. of a quad. eq. with real coefficients, what is the other sol.?

\( 2-3i \) since complex conjugate

ex.) Find the zeroes. \( x^4 + 5x^2 + 4 = 0 \)

use u-substitution. Let \( u = x^2 \)

\[ u^2 + 5u + 4 = 0 \]

\[ (u + 4)(u + 1) = 0 \]

\[ u + 4 = 0 \quad \text{or} \quad u + 1 = 0 \]

\[ u = -4 \quad \text{or} \quad u = -1 \]

\[ x^2 = -4 \quad \text{or} \quad x^2 = -1 \]

\( x = \pm 2i \quad \text{or} \quad x = \pm i \)
§4.10 - Nothing New

1. Degree = # of complex zeroes
2. Rational Root Theorem

ex. \( f(x) = x^4 + 6x^3 + 11x^2 + 12x + 18 \)
   a. Degree 4 \( \Rightarrow \) 4 zeroes
   b. RRT: \( p = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18 = \pm \frac{P}{Q} \) = potential zeroes

\[ q = \pm 1 \]

\[ f(1) = 1 + 6 + 11 + 12 + 18 = 48 \neq 0 \]
\[ f(-1) = -1 - 6 + 11 - 12 + 18 = 12 = 0 \]
\[ f(2) = 16 + 48 + 44 - 24 + 18 = 60 \neq 0 \]
\[ f(-3) = 81 - 158 + 99 - 36 + 18 = 0 \sqrt{50} \] (x + 3) is a factor

\[ \frac{1}{3} \quad 1 \quad 6 \quad 11 \quad 12 \quad 18 \]
\[ \quad \downarrow \quad -3 \quad -9 \quad -6 \quad -18 \]
\[ \quad 1 \quad 3 \quad 2 \quad 6 \quad 0 \]

\[ x^3 + 3x^2 + 2x + 6 \] ... factor or RRT again

\[ x^2(x + 3) + 2(y + 3) \]
\[ (x + 3)(x^2 + 2) \]
\[ x + 3 = 0 \]
\[ x = -3 \] again
\[ x^2 + 2 = 0 \]
\[ x = \pm \sqrt{2i} \]
\[ x = \pm 2i \] or \( \frac{3}{2} \) and \( -\frac{3}{2} \) each of mult 1

Zeroes are \(-3, -3, \sqrt{2i}, -\sqrt{2i}\)

\( f(x) \) factored is \((x + 3)(x + 3)(x + \sqrt{2i})(x - \sqrt{2i})\)

or \((x + 3)^2(x + \sqrt{2i})(x - \sqrt{2i})\)