September 20, 2012

§6.4

Graphing Sine and Cosine Functions:

\[ y = A \sin (wx) \quad y = A \cos (wx) \]

- Amplitude: \( A \) (vertical dist. from peak to trough)
- Period: \( \frac{2\pi}{w} \) (horizontal dist. from peak to peak or trough to trough)

### Graphs

1. \( y = \sin x \)
2. \( y = \cos x \)

### Graph Analysis

- **Domain:** \( \mathbb{R} \)
- **Range:** \([-1, 1]\)

Thus, \( \sin x = \cos (x - \frac{\pi}{2}) \).

### Steps for Graphing Sine and Cosine

1. Find amplitude and period.
2. Divide \([0, \frac{2\pi}{w}]\) into \(8\) equal subintervals.
3. Use endpoints of subintervals to find key points.
4. Plot key points and connect the dots!

### Example (12 from WS)

\[ y = -2 \cos \left( \frac{\pi}{2} x \right) \]

- **Amplitude:** \( |A| = 2 \)
- **Period:** \( \frac{2\pi}{w} = \frac{2\pi}{\frac{\pi}{2}} = 4 \)

### Correspondences

- \( \frac{\pi}{2} \)
- \( \frac{3\pi}{2} \)
- \( \frac{5\pi}{2} \)
- \( \frac{7\pi}{2} \)
§6.5

- graphs of $\tan$, $\cot$, $\sec$, $\csc$: functions
  $y = A \tan(bx)$  $y = A \cot(bx)$
  $|A|$ = magnitude (of vertical stretch)
  $T = \frac{\pi}{b}$ = period for $\tan$, $\cot$
  $L$ = horizontal compression/stretch by factor of $\frac{1}{b}$

$y = \tan x$

$y = \cot x$

Steps for graphing tangent and cotangent:
1. Find magnitude and period
2. Draw in dotted vertical lines as asymptotes
3. Draw in key points on x-axis & midpoints between asymptotes
4. Plot other key points (when $y = \pm A$) to complete graph.
   ex.) ($\pm 14$ from WS)

$y = 3 \cot \left(\frac{x}{2}\right) - 2$

Magnitude $= |3| = 3$
Period $= \frac{\pi}{b} = \frac{\pi}{\frac{1}{2}} = 2\pi$
Vertical shift $= -2$ (down)
36.5 (cont.)

\[ y = A \csc (wx) \quad y = A \sec (wx) \]

1. \( |A| \): magnitude of vertical stretch (sets range)
2. \( T = \frac{2\pi}{|w|} \): period
3. \( \rightarrow \) non-horizontal compression/stretch = \( \frac{1}{|b|} \)

**Graph of \( y = \csc x \)**

- Domain: \( \mathbb{R} \pm k\pi; \ k = 1, 2, 3 \)
- Range: \( (-\infty, -1] \cup [1, \infty) \)

**Graph of \( y = \sec x \)**

- Domain: \( \mathbb{R} \pm \frac{\pi}{2}; \ k = 1, 2, 3 \)
- Range: \( (-\infty, -1] \cup [1, \infty) \)

**Steps for graphing cosecant and secant**

1. Find magnitude and period
2. Stretch corresponding sine/cosine graph
3. Draw in dotted vertical lines as asymptotes
4. Plot key points when \( y = \pm A \) (at min./max. of parabolas)
5. Draw in parabolas to complete graph.

**Example:** \( f(x) = 2\sec (4x) + 1 \)

- Magnitude = \( |2| = 2 \)
- Period = \( \frac{\pi}{2} = \frac{\pi}{4} \)
- Vertical shift up = 1

**Graph:**

- Key points and asymptotes are marked on the graph.