PARABOLAS

1.) What is the equation of a parabola with its vertex at (0,0) and focus at (a,0)?

\[ y^2 = 4ax \]

2.) What is the equation of a parabola with its vertex at (h, k) and focus at (h+a, k)?

\[ (y-k)^2 = 4a(x-h) \]

3.) Find the equation of the parabola with its focus at (0, -1) and its directrix as the line \( y = 1 \).

\[ x^2 = 4ay \]

4.) Find the equation of the parabola with its vertex at (0,0), its axis of symmetry as the x-axis, and containing the point (2,3).

\[ y^2 = 4a x \]

5.) Find the vertex, focus, and directrix of each parabola.

a.) \((x+4)^2 = 16(y+2)\)

- **Vertex**: (-4, -2)
- **Focus**: (-4, -2 + \( \frac{10}{4} \)) = (-4, 2)
- **Directrix**: \( y = -2 - \frac{10}{4} \) or \( y = -6 \)

b.) \( x^2 - 4x = y + 4 \)

\( (x-2)^2 = y + 8 \)

- **Vertex**: (2, -3)
- **Focus**: (2, -3 + \( \frac{1}{4} \)) = (2, -\( \frac{23}{4} \))
- **Directrix**: \( y = -3 - \frac{1}{4} \) or \( y = -\frac{13}{4} \)
ELLIPSES

1.) What is the equation of an ellipse centered at \((h, k)\) with its major axis parallel to the x-Axis?

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

2.) What is the equation of an ellipse centered at \((h, k)\) with its major axis parallel to the y-Axis?

\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1
\]

3.) Find the equation for the ellipse with foci at \((\pm 2, 0)\) and a major axis of length 6.

\[
c = 2 = \sqrt{a^2 - b^2}
\]
\[
2 = \sqrt{9 - b^2}
\]
\[
4 = 9 - b^2
\]
\[
-5 = -b^2
\]
\[
\pm \sqrt{5} = b
\]

\[
\frac{x^2}{9} + \frac{y^2}{5} = 1
\]

4.) Find the center, foci, and vertices of each ellipse.

a.) \[
\frac{(x+4)^2}{9} + \frac{(y+2)^2}{4} = 1
\]
   Center: \((-4, -2)\)
   Foci: \(c^2 = a^2 - b^2\)
   \(c = \pm \sqrt{9 - 4}\)
   \(c = \pm 1\)
   \((-4 \pm 1, -2)\)
   Vertices: \((-4 + 3, -2)\) and \((-4 - 3, -2)\)

b.) \[
2x^2 + 3y^2 - 8x + 6y + 5 = 0
\]
   \(2(x^2 - 4x + 4) + 3(y^2 + 2y + 1) = -5 + 2(4) + 3(1)\)
   \(2(x-2)^2 + 3(y+1)^2 = 1\)
   Center: \((2, -1)\)
   Foci: \(c^2 = a^2 - b^2\)
   \(c = \pm \sqrt{3-2} = \pm 1\)
   \((1, 1)\) and \((3, -1)\)
   Vertices: \((2 \pm \sqrt{3}, -1)\)

5.) Find an equation for the ellipse centered at \((-3, 1)\) with a vertex at \((-3, 3)\) and a focus at \((-3, 0)\).

\[
c^2 = a^2 - b^2
\]
\[
1 = \sqrt{4 - b^2}
\]
\[
-3 = -b^2
\]
\[
\pm \sqrt{3} = b
\]

\[
\frac{(x+3)^2}{3} + \frac{(y-1)^2}{4} = 1
\]

6.) Find the equation for the ellipse centered at \((1, 2)\) with a focus at \((1, 4)\) a containing the point \((2, 2)\).

\[
c = 2 = \sqrt{a^2 - b^2}
\]
\[
4 = a^2 - 1
\]
\[
\pm \sqrt{3} = a
\]

\[
\frac{(x-1)^2}{1} + \frac{(y-2)^2}{5} = 1
\]
HYPERBOLAS

1.) What is the equation of a hyperbola centered at \((h, k)\) with the transverse axis parallel to the x-axis? What are the equations of its asymptotes?
\[
\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1
\]
\[
y - k = \pm \frac{b}{a} (x - h)
\]

2.) What is the equation of a hyperbola centered at \((h, k)\) with the transverse axis parallel to the y-axis? What are the equations of its asymptotes?
\[
\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1
\]
\[
y - k = \pm \frac{a}{b} (x - h)
\]

3.) Find the equation for the hyperbola with vertices at \((0, -5)\) and \((0, 6)\) and one asymptote of \(y = 2x\).
\[
\frac{x^2}{36} - \frac{y^2}{9} = 1
\]
\[
a = \frac{|0| + |0|}{2} = 0
\]
\[
\text{center @ } (0,0)
\]
\[
y = 2x = \frac{6}{6} x
\]
\[
\Rightarrow b = \pm 3
\]

4.) Find the equation for the hyperbola with its foci at \((3, 7)\) and \((7, 7)\) and a vertex at \((6, 7)\).
\[
\text{center @ } (\frac{3+7}{2}, 7) = (5, 7)
\]
\[
\frac{(x-5)^2}{1} - \frac{(y-7)^2}{3} = 1
\]
\[
a = b - 5 = 1
\]
\[
b^2 = 4 - 1 = 3
\]
\[
b = \pm \sqrt{3}
\]

5.) Find the equation for the hyperbola with its vertices at \((1, -3)\) and \((1, 1)\) and the asymptote: \(y + 1 = \frac{3}{2} (x - 1)\).
\[
\text{center @ } (1, \frac{-3+1}{2}) = (1, -1)
\]
\[
\frac{(y+1)^2}{4} - \frac{9(x-1)^2}{16} = 1
\]
\[
\frac{3}{2} = \frac{a}{b} \text{ and } a = |1 - 1| = 2
\]
\[
\Rightarrow b = \frac{4}{3}
\]

6.) Find the center, transverse axis, vertices, foci, and asymptotes.
   a.) \((x + 4)^2 - 9(y - 3)^2 = 9\)
   \[
   \frac{(x+4)^2}{9} - \frac{(y-3)^2}{1} = 1 \Rightarrow c = \sqrt{10}
   \]
   \[
   \text{center @ } (-4, 3)
   \]
   \[
   \text{transverse axis: } y = 3
   \]
   \[
   \text{vertices: } (-1, 3) \text{ and } (-7, 3)
   \]
   \[
   \text{foci: } (-4 + \sqrt{10}, 3) \text{ and } (-4 - \sqrt{10}, 3)
   \]
   \[
   \text{asymptotes: } y - 3 = \pm \frac{3}{1}(x + 4)
   \]
   b.) \(y^2 - 4x^2 - 16x - 2y - 19 = 0\)
   \[
   \frac{(y+2)^2}{1} - \frac{4(x^2 + 4x + 4)}{4} = 19 + 16 - 4\cdot4
   \]
   \[
   \frac{(y+2)^2}{1} - (x+2)^2 = 1 \Rightarrow a = 1 \Rightarrow c = \sqrt{5}
   \]
   \[
   \text{center @ } (-2, 1)
   \]
   \[
   \text{transverse axis: } x = -2
   \]
   \[
   \text{vertices: } (-2, 3) \text{ and } (-2, -1)
   \]
   \[
   \text{foci: } (-2, 1 + \sqrt{5}) \text{ and } (-2, 1 - \sqrt{5})
   \]
   \[
   \text{asymptotes: } y + 1 = \pm 2(x + 2)
   \]