Math 1151
Fall 2004 Final Exam Problems

This exam contains 19 written problems, worth 10 to 20 points each, for a total of 270 points.

1. (15 pts) If \( \sin(\theta) = \frac{12}{13} \) and \( \theta \) is acute, find \( \cos(\theta) \), \( \tan(\theta) \), \( \sin(2\theta) \), \( \cos(2\theta) \).

2. (10 pts) Prove that \( 1 + \tan^2(\theta) = \sec^2(\theta) \). (You may use the fact that \( \sin^2(\theta) + \cos^2(\theta) = 1 \).

3. (10 pts) Solve the system
\[
\begin{align*}
x + y &= 1000 \\
x - y &= 8
\end{align*}
\]

4. (10 pts) Express the point \((\sqrt{3},1)\) in polar coordinates.

5. (10 pts) Evaluate the product \((3 + 4i)(5 + 6i)\).

6. (15 pts) Establish the identity
\[
\cos(\alpha - \beta) = \cot(\alpha) + \tan(\beta)
\]
7. (15 pts) Solve the system

\[
\begin{align*}
  y &= 3x - 5 \\
  x^2 + y^2 &= 5.
\end{align*}
\]

Provide all solutions.

8. (15 pts) Prove that \(\cot(2\theta) = \frac{\cot^2(\theta) - 1}{2 \cot \theta}\)

9. (20 pts) In \(\triangle ABC\), \(a = 3\), \(b = 4\), angle \(C = \gamma = 30^\circ\). Find c. Find \(\sin A = \sin(\alpha)\) and \(\sin(B) = \sin(\beta)\). Find the area of \(\triangle ABC\).

10. (10 pts) Find a polynomial of degree 4 with real coefficients having roots 1, -1, and \(-i\). Express your answer as a polynomial in real coefficients.

11. (15 pts) Evaluate the sum

\[
1 + 3 + 5 + 7 + 9 + \cdots + 1001
\]

12. (15 pts) Find the sum of the series

\[
2 + 2^2 + \cdots + 2^{64}.
\]

13. (15 pts) Find the sum of the infinite geometric series

\[
\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots
\]
14. (15 pts) Find the center of the ellipse
\[ 4x^2 + y^2 - 8x + 4y + 4 = 0 \]

15. (15 pts) Find all real and complex values of \( x \) for which \( x^3 - 1 = 0 \).

16. (15 pts) Prove that \( (x + 1) \) is a factor of \( x^{49} + 1 \).

17. (15 pts) Prove that \( \sin(3\theta) = 3\sin \theta - 4\sin^3 \theta \). (Hint: write \( 3\theta \) as \( 2\theta + \theta \)).

18. (20 pts) Find all values of \( \theta \) for which
\[ \sin \theta - \sqrt{3} \cos \theta = 2. \]

19. (15 pts) Find the square root of the complex number
\[ \frac{1}{2} + i\frac{\sqrt{3}}{2} \]