Center-Manifold Reduction for Spiral Waves

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Abstract - Spiral waves are rotating waves of reaction-diffusion equations on the plane. In this note, a center-manifold reduction for dynamics of spiral waves is presented. Bifurcations of rigidly-rotating spiral waves are then described by ordinary differential equations, which are equivariant under the (special) Euclidean group $SE(2)$. Several difficulties arise in the analysis since $SE(2)$ is not compact and does not induce a strongly continuous group action on the underlying function space.

Réduction à une Variété Centrale pour Ondes Spirales

Résumé - Les ondes spirales sont des ondes rotatives d'équations réaction-diffusion dans le plan. Dans cette note, nous présentons une réduction à une variété centrale pour la dynamique d'ondes spirales. Des bifurcations à partir d'ondes spirales sont alors décrites par des équations différentielles ordinaires, équivariantes sous le groupe euclidien $SE(2)$. Plusieurs difficultés apparaissent car le groupe de symétrie n'est pas compact et n'agit pas d'une façon fortement continue sur l'espace fonctionel décrivant le problème.

Version française abrégée - Les ondes spirales représentent un phénomène typique en création de structure spatiale, observées dans divers systèmes biologiques, chimiques ou physiques. La dynamique de la plupart de ces systèmes peut être décrite par des équations réaction-diffusion (1). La matrice $D = \mathrm{diag}(d_j)$ est diagonale et l'on suppose que $d_j \geq 0$. La fonction $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ est supposée C^∞. L'équation (1) est équivariante sous les symétries euclidiennes $SE(2)$ du plan agissant sur le domaine $\mathbb{R}^2$. Pour $(a, \varphi) \in SE(2) = \mathbb{R}^2 \times S^1$, l'on a $(T_{a,\varphi}u)(x) := u(R_{-\varphi}(x-a))$, où la transformation R_φ désigne la rotation par l'angle φ autour de zéro dans $\mathbb{R}^2$. L'équation (1) génère un semi-flot lisse $\Phi_t(u, \mu)$ dans l'espace $C^0_{\mathrm{unif}}(\mathbb{R}^2, \mathbb{R}^n)$ de fonctions bornées et uniformément continues. Notons pourtant que l'action de la rotation sur cet espace n'est pas continue.

Mathématiquement, une onde spirale $u_* \in C^0_{\mathrm{unif}}(\mathbb{R}^2, \mathbb{R}^n)$ est une onde rotative : $\Phi_t(u_*, \mu_*) = T_{0,\omega_* t} u_*$. Nous nous intéressons à la transition d'ondes rigidement rotatives vers des ondes méandrantes ou propageantes. Les deux phénomènes sont générés par une bifurcation de Hopf à partir d'ondes spirales. Cette transition a été étudiée numériquement par Barkley [1] et des équations modèles ont été proposées [2]. Wulff [6] a obtenu des résultats rigoureux sur la bifurcation de Hopf via une réduction de type Lyapunov-Schmidt.

Dans cette note, est établi un théorème sur l'existence d'une variété centrale dans le cadre d'un semi-flot équivariant. La dynamique stable proche d'une onde rotative est réduite à des équations différentielles équivariantes sur une variété de dimension finie. Les problèmes majeurs proviennent de l'action non compacte et non continue du groupe $SE(2)$.

Nous considérons l'équation (2) dans un espace de Banach Y. Nous supposons A, sectoriel et $F : Y^\alpha \times \mathbb{R}^p \to Y$, lisse pour un $\alpha \in [0, 1)$ (voir Henry [4] pour la notation).

Supposons que (2) est équivariant par une représentation du groupe $SE(2)$ dans $GL(Y^\alpha)$. On désigne par $\Phi_t(u, \mu)$ le semi-flot dans Y^α, associé à (2). On travaillera toujours dans les espaces

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d'interpolation Y^α avec les normes $|\cdot|$ et $\|\cdot\|$ pour vecteurs et opérateurs.

On suppose que l'action du groupe est bornée et faiblement continue :

Hypothèse 1 *Il existe $K > 0$ tel que $\|T_{a,\varphi}\| \leq K$ pour tout $T_{a,\varphi} \in SE(2)$. En outre, si $T_{0,\varphi_n} u \to v$ dans Y^α, alors $u = v$.*

Supposons maintenant qu'une onde rotative existe :

Hypothèse 2 *Il existe $u_* \in Y^\alpha$, $\mu_* \in \mathbb{R}^p$ et $\omega_* \in \mathbb{R}$ avec $\omega_* \neq 0$ tel que $\Phi_t(u_*, \mu_*) = T_{0,\omega_* t} u_*$ pour tout t.*

Nous obtenons alors $\Phi_{2\pi/\omega_*}(u_*, \mu_*) = u_*$.

Hypothèse 3 *L'ensemble $\operatorname{spec}(D\Phi_{2\pi/\omega_*}(u_*, \mu_*)) \cap \{\lambda \in \mathbb{C}; |\lambda| \geq 1\}$ est un ensemble spectral avec un sous-espace propre généralisé E_*^{cu} de dimension $m < \infty$.*

Hypothèse 4 *(i) La fonction $a \mapsto T_{a,0}(u_* + v)$ est lisse entre $\mathbb{R}^2$ et Y^α pour chaque $v \in E_*^{cu}$.*

(ii) Pour tout $\epsilon > 0$, il existe $\delta > 0$ tel que $|T_{a,\varphi} u_* - u_*| \geq \delta$ pour tout $T_{a,\varphi} \in SE(2)$ avec $\|T_{a,\varphi} - T_{\hat{a},\hat{\varphi}}\| \geq \epsilon$ pour tout $T_{\hat{a},\hat{\varphi}} \in \Sigma_*$.

Les deux hypothèses précédentes garantissent que l'orbite du groupe $SE(2)$, avec les sous-espaces E_*^{cu}, est lisse.

Théorème 1 *Supposons les hypothèses 1 – 4. Fixons un nombre $k \in \mathbb{N}$. Alors pour tout μ avec $|\mu - \mu_*|$ suffisamment petit, il existe une variété de dimension m dans Y^α, invariant sous le groupe $SE(2)$ et localement invariant sous le semi-flot. La variété M_μ^{cu} et la dépendance des paramètres sont de la classe C^k. De plus, M_μ^{cu} contient toutes solutions qui restent dans un voisinage suffisamment petit de l'orbite de groupe de u_* pour tous temps négatifs. Finalement la variété M_μ^{cu} est localement exponentiellement attractive.*

Considérons l'espace $\mathbb{C} \times S^1 \times V_*$ avec l'action de $SE(2)$

$$T_{\tilde{a},\tilde{\varphi}}(a, \varphi, v) = (e^{i\tilde{\varphi}} a + \tilde{a}, \varphi + \tilde{\varphi}, v).$$

où $(a, \varphi) \in \mathbb{C} \times S^1$ et $v \in V_*$. Le sous-groupe d'isotropie $\mathbb{Z}_l$ de u_* agit sur $\mathbb{C} \times S^1 \times V_*$ selon $S_{\hat{\varphi}}(a, \varphi, v) := (e^{i\hat{\varphi}} a, \varphi, T_{0,\hat{\varphi}} v)$.

Théorème 2 *Supposons que le sous-groupe d'isotropie de u_* est fini. Sous les hypothèses 1 – 4, la variété M_μ^{cu} est difféomorphe à $(\mathbb{C} \times S^1 \times V_*)/\!\sim$, où la relation d'équivalence $\sim$ est définie par les orbites de groupe de l'isotropie $S_{\hat{\varphi}}\, T_{0,-\hat{\varphi}}$, c'est-à-dire,*

$$(a, \varphi, v) \sim S_{\hat{\varphi}}\, T_{0,-\hat{\varphi}}\,(a, \varphi, v) = (a, \varphi - \hat{\varphi}, T_{0,\hat{\varphi}} v)$$

pour $(0, \hat{\varphi}) \in \Sigma_*$. Le champ vectoriel sur M_μ^{cu} est donné par (3), et $G(a, \varphi, v, \mu) \in \mathbb{C} \times \mathbb{R} \times V_*$ est C^k et équivariant sous l'action $S_{\hat{\varphi}}$ de Σ_*, voir (4). De plus, nous avons $G(a, \varphi, 0, \mu_*) = (0, \omega_*, 0)$.

Le théorème 2 garantit que les bifurcations à partir d'ondes spirales sont déterminées par g_N et la propagation est gouvernée par $g_T = DT_{a,\varphi}(g_1, g_2)(v, \mu)$.

Si l'on suppose qu'une onde spirale $u_* \in Y = C^0_{\text{unif}}(\mathbb{R}^2, \mathbb{R}^n)$ de l'équation 1 vérifie les hypothèses 2 et 3 et si $d_j > 0$, l'hypothèse 4 est aussi vérifiée (voir Lemma 1) ; les théorèmes peuvent être alors appliqués. En outre, le sous-groupe d'isotropie de u_* ne peut être que $SE(2)$, S^1, ou $\mathbb{Z}_\ell$. Dans les deux premiers cas u_* est effectivement une onde stationnaire.

En fait, l'hypothèse principale,l'hypothèse 2 sur le spectre,peut être vérifiée en considérant la linéarisation dans des coordonnées rotatives (5).

1 Introduction

Spiral waves are a typical phenomenon of spatio-temporal pattern formation. They are observed in various biological, chemical, and physical systems, for instance, in the Belousov-Zhabotinsky reaction, the catalysis on platinum surfaces, and the Rayleigh-Benard convection.

The dynamics of such systems is governed by reaction-diffusion equations

$$u_t = D\Delta u + f(u,\mu), \qquad\qquad x \in \mathbb{R}^2, \tag{1}$$

on the plane. Here, $D = \mathrm{diag}(d_j)$ is a diagonal matrix with non-negative entries $d_j \geq 0$, and $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ is smooth, that is, C^∞. Equation (1) is equivariant under the Euclidean symmetry $SE(2)$ of the plane acting on the domain. Hence, for $(a,\varphi) \in SE(2) = \mathbb{R}^2 \times S^1$, we have $(T_{a,\varphi}u)(x) := u(R_{-\varphi}(x-a))$ where the matrix R_φ denotes the rotation by the angle φ around zero in $\mathbb{R}^2$. Equation (1) is well-posed on the space $C^0_{\mathrm{unif}}(\mathbb{R}^2,\mathbb{R}^n)$ of uniformly continuous, bounded functions and generates a smooth semiflow denoted by $\Phi_t(u,\mu)$. Note, however, that the rotations $T_{0,\varphi}$ do not act as a strongly continuous semigroup on C^0_{unif}.

We are interested in the transition from rigidly-rotating to meandering and drifting spiral waves. This transition has been investigated experimentally as well as numerically in the systems mentioned above, see [6] for appropriate references. In mathematical terms, rigidly-rotating spiral waves are rotating waves, that is, they obey $\Phi_t(u_*,\mu_*) = T_{0,\omega_* t}u_*$. Thus, they are equilibria in a frame rotating with the frequency ω_* of the rotating wave. We shall further distinguish two kinds of modulated waves. Meandering spiral waves are quasiperiodic solutions which are periodic in a rotating frame. In contrast, drifting spiral waves are periodic in a moving frame. Both, meandering and drifting spiral waves, emanate from rigidly-rotating spiral waves by Hopf bifurcations in a rotating frame. This has been verified numerically by Barkley [1]. Barkley [2] proposed a five-dimensional system of ordinary differential equations modeling the qualitative behavior of reaction-diffusion systems near Hopf bifurcations from rotating waves. However, a rigorous relation between the two systems has not been established yet.

Wulff [6] investigated Hopf bifurcations from rotating to meandering and drifting spiral waves using Lyapunov-Schmidt reduction in the largest subspace of C^0_{unif} on which the rotations $T_{0,\varphi}$ act as a strongly continuous semigroup.

In this article, we formulate a center-manifold theorem in an abstract $SE(2)$-equivariant setting. The dynamics in the vicinity of a rotating wave is reduced to ordinary differential equations. In particular, the drift along the group orbit as well as bifurcations in the directions normal to it are accessible to a dynamical-systems approach. The difficulties arising in the analysis are the non-compactness of the group $SE(2)$ and the lack of continuity of the group action on C^0_{unif}.

2 Results

Consider a semilinear differential equation

$$u_t = -Au + F(u,\mu), \tag{2}$$

on some Banach space Y. We assume that A is sectorial and F is a smooth function from $Y^\alpha \times \mathbb{R}^p$ to Y for some $\alpha \in [0,1)$, see Henry [4] for the notation. Suppose that (2) is equivariant with respect to a representation of $SE(2)$ in the group of linear, bounded and invertible operators on Y^α. The semiflow on Y^α associated with (2) is denoted by $\Phi_t(u,\mu)$. Throughout, we shall work within the space Y^α. The norms $|\cdot|$ and $||\cdot||$ for vectors and operators are those induced by Y^α.

The following weak-continuity and boundedness condition is imposed on the group action.

Hypothesis 1 *There exists a constant K such that $\|T_{a,\varphi}\| \leq K$ for all $T_{a,\varphi} \in SE(2)$ where we identify group elements with their representation. Furthermore, if $T_{0,\varphi_n}u$ converges to v in Y^α for some sequence $\varphi_n \to 0$, then $u = v$ holds.*

Next, suppose that a rotating-wave solution of (2) exists.

Hypothesis 2 *There exist $u_* \in Y^\alpha$, $\mu_* \in \mathbb{R}^p$, and $\omega_* \in \mathbb{R}$ with $\omega_* \neq 0$ such that $\Phi_t(u_*, \mu_*) = T_{0,\omega_* t}u_*$ holds for all t.*

In particular, $\Phi_{2\pi/\omega_*}(u_*, \mu_*) = u_*$ holds. We assume the following hypothesis on the spectrum of $D\Phi_{2\pi/\omega_*}(u_*, \mu_*)$.

Hypothesis 3 *The set $\mathrm{spec}(D\Phi_{2\pi/\omega_*}(u_*, \mu_*)) \cap \{\lambda \in \mathbb{C}; |\lambda| \geq 1\}$ is a spectral set with generalized eigenspace E_*^{cu} of finite dimension $m < \infty$.*

As $SE(2)$ is not compact and may not act continuously on Y^α, we introduce another assumption. Let Σ_* denote the isotropy subgroup of the rotating wave u_*.

Hypothesis 4 *(i) The function $a \mapsto T_{a,0}(u_* + v)$ from $\mathbb{R}^2$ to Y^α is smooth for any fixed $v \in E_*^{cu}$.*

(ii) For any $\epsilon > 0$, there exists a $\delta > 0$ such that $|T_{a,\varphi}u_* - u_*| \geq \delta$ for all $T_{a,\varphi} \in SE(2)$ with $\|T_{a,\varphi} - T_{\hat{a},\hat{\varphi}}\| \geq \epsilon$ for all $T_{\hat{a},\hat{\varphi}} \in \Sigma_*$.

Hypothesis 4 ensures that the group orbit $SE(2)u_*$ of u_* is a smooth embedded manifold. Now we state the main result.

Theorem 1 *Assume that Hypotheses 1 – 4 are obeyed. Fix some number $k \in \mathbb{N}$. Then, for any μ with $|\mu - \mu_*|$ sufficiently small, there exists an $SE(2)$-invariant, locally semiflow-invariant, m-dimensional manifold M_μ^{cu} contained in Y^α. The manifold M_μ^{cu} is of class C^k and depends C^k-smoothly on the parameter μ. Furthermore, M_μ^{cu} contains all solutions which stay close to the group orbit of u_* for all negative times. Moreover, M_μ^{cu} is locally exponentially attracting.*

The flow on M_μ^{cu} can be characterized as follows. The vector space $E_*^{cu} = \mathcal{T}_{u_*}(SE(2)u_*) \oplus V_*$ splits into two Σ_*-invariant subspaces. Consider the space $\mathbb{C} \times S^1 \times V_*$ with the $SE(2)$-multiplication

$$T_{\tilde{a},\tilde{\varphi}}(a, \varphi, v) = (e^{\mathrm{i}\tilde{\varphi}}a + \tilde{a}, \varphi + \tilde{\varphi}, v).$$

where $(a, \varphi) \in \mathbb{C} \times S^1$ and $v \in V_*$. Define the linearized operator $DT_{\tilde{a},\tilde{\varphi}}(\xi_a, \xi_\varphi) = (e^{\mathrm{i}\tilde{\varphi}}\xi_a, \xi_\varphi)$ for $(\xi_a, \xi_\varphi) \in \mathbb{C} \times \mathbb{R}$. Equivariant vector fields on $\mathbb{C} \times S^1 \times V_*$ are given by

$$\begin{pmatrix} \dot{a} \\ \dot{\varphi} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} e^{\mathrm{i}\varphi}\, g_1(v,\mu) \\ g_2(v,\mu) \\ g_N(v,\mu) \end{pmatrix} = \begin{pmatrix} DT_{a,\varphi} \begin{pmatrix} g_1(v,\mu) \\ g_2(v,\mu) \end{pmatrix} \\ g_N(v,\mu) \end{pmatrix} = G(a, \varphi, v, \mu) \tag{3}$$

for $v \in V_*$ with $|v| < \delta$. Suppose that the isotropy subgroup $\Sigma_* = \mathbb{Z}_\ell$ of u_* is finite. Then $(0, \hat{\varphi}) \in \Sigma_*$ acts on $\mathbb{C} \times S^1 \times V_*$ according to $S_{\hat{\varphi}}(a, \varphi, v) := (e^{\mathrm{i}\hat{\varphi}}a, \varphi, T_{0,\hat{\varphi}}v)$.

Theorem 2 *Assume that Hypotheses 1 – 4 hold. In addition, suppose that the isotropy subgroup Σ_* of u_* is finite. Then the manifold M_μ^{cu} is diffeomorphic to $(\mathbb{C} \times S^1 \times V_*)/\sim$ where the equivalence relation is defined by group orbits of the Σ_*-action $S_{\hat{\varphi}}\, T_{0,-\hat{\varphi}}$, that is,*

$$(a, \varphi, v) \sim S_{\hat{\varphi}}\, T_{0,-\hat{\varphi}}\,(a, \varphi, v) = (a, \varphi - \hat{\varphi}, T_{0,\hat{\varphi}}v)$$

for $(0,\hat{\varphi}) \in \Sigma_*$. *The vector field on* M_μ^{cu} *is given by (3) modulo the above equivalence relation. The nonlinearity* $G(a,\varphi,v,\mu) \in \mathbb{C} \times \mathbb{R} \times V_*$ *is* C^k *and is equivariant under the action* $S_{\hat{\varphi}}$ *of* Σ_**, that is,*

$$\begin{pmatrix} e^{\mathrm{i}\varphi}\, g_1(T_{0,\hat{\varphi}}v,\mu) \\ g_2(T_{0,\hat{\varphi}}v,\mu) \\ g_N(T_{0,\hat{\varphi}}v,\mu) \end{pmatrix} = \begin{pmatrix} e^{\mathrm{i}(\varphi+\hat{\varphi})}\, g_1(v,\mu) \\ g_2(v,\mu) \\ T_{0,\hat{\varphi}}\, g_N(v,\mu) \end{pmatrix} \tag{4}$$

for all $(0,\hat{\varphi}) \in \Sigma_*$. *Moreover,* $G(a,\varphi,0,\mu_*) = (0,\omega_*,0)$.

Conversely, any vector field G *with the above properties yields a well-defined vector field on* M_μ^{cu} *after factoring by the equivalence relation.*

Theorem 2 asserts that bifurcations of spiral waves are determined by bifurcations of the Σ_*-equivariant normal vector field g_N

$$\dot{v} = g_N(v,\mu), \qquad\qquad v \in V_* \cong \mathbb{R}^{m-3},$$

on V_*, while the drift along the group orbit of the spiral wave is governed by the tangent vector field $g_T = DT_{a,\varphi}(g_1,g_2)(v,\mu)$ on $SE(2)$. Similar results for smooth actions of compact groups have been proved by Krupa [5]. The effects of Hopf bifurcation from relative equilibria with compact isotropy in the presence of general noncompact symmetry groups, in particular various kinds of drifting and meandering spiral waves in the presence of $SE(3)$, are discussed in [3].

3 Applications to reaction-diffusion systems

We return to a discussion of the reaction-diffusion system (1)

$$u_t = D\Delta u + f(u,\mu), \qquad\qquad x \in \mathbb{R}^2,$$

on the plane, see Section 1, and consider (1) on the space $Y = C^0_{\mathrm{unif}}(\mathbb{R}^2,\mathbb{R}^n)$ with $\alpha = 0$. Note that we allow for vanishing diffusion coefficients in the matrix $D = \mathrm{diag}(d_j)$. We observe that Hypothesis 1 is obeyed by the action of $SE(2)$ on the space C^0_{unif} given in Section 1.

Lemma 1 *Suppose that* $u_* \in C^0_{\mathrm{unif}}$ *is a rotating wave of (1) satisfying Hypotheses 2 and 3. If* $d_j = 0$ *for some* j*, we assume in addition that* u_* *and any* $v \in E_*^{cu}$ *are smooth functions. Then Hypothesis 4 holds as well. Moreover, the isotropy subgroup of* u_* *is isomorphic to* $SE(2)$, S^1, *or* $\mathbb{Z}_\ell$ *for some finite* ℓ. *In the first two cases,* u_* *is actually a standing wave.*

With Lemma 1 at hands, it suffices to verify Hypotheses 2 and 3 for an application of the theorems stated in the last section. As Hypothesis 2 merely states existence of spiral waves, the major assumption made is on the spectrum of the linearization of the period map $D\Phi_{2\pi/\omega_*}(u_*,\mu_*)$. However, it is possible to write this assumption in terms of the spectrum of the operator

$$L := D\Delta - \omega_* \frac{\partial}{\partial\varphi} + D_u f(u_*,\mu_*), \tag{5}$$

that is, the linearization of the spiral wave in a rotating frame. Note that L generates a C^0-semigroup on the space $C^0_{\mathrm{eucl}}(\mathbb{R}^2,\mathbb{R}^n) := \mathrm{clos}_{C^0_{\mathrm{unif}}} D(\frac{\partial}{\partial\varphi})$, that is, the closure of the domain of the generator of rotations in the space $C^0_{\mathrm{unif}}(\mathbb{R}^2,\mathbb{R}^n)$, see [6].

Lemma 2 *Consider equation (1) on the space* $C^0_{\mathrm{eucl}}(\mathbb{R}^2,\mathbb{R}^n)$. *Furthermore, assume that* u_* *obeys Hypothesis 2. Suppose that* $\mathrm{spec}(L) \cap \{\lambda \in \mathbb{C}; \mathrm{Re}\,\lambda \geq 0\}$ *is a spectral set with spectral projection* P_*. *If* $\dim P_* C^0_{\mathrm{eucl}} < \infty$ *and the semigroup* e^{Lt} *satisfies*

$$\| e^{Lt}|_{(1-P_*)C^0_{\mathrm{eucl}}} \| \leq M e^{-\beta t}$$

for some $\beta > 0$, *then Hypothesis 3 is true.*

We emphasize that $L|_{E_*^{cu}}$ determines the linearization $D_v g_N(0,\mu_*)$ of the normal vector field.

4 Outline of the proof

Main technique for proving Theorem 1 is the graph transform applied to $\Phi_T(u, \mu)$ for some large time T. Hypothesis 3 guarantees the existence of smooth, equivariant center-unstable and stable bundles over the group orbit $SE(2)u_*$ using, for instance, Dunford integrals. In order for the graph transform to work, the vector field has to be modified in the center directions only. If the modified vector field were smooth and equivariant, a standard argument would show that the resulting invariant center-unstable manifold is also equivariant. Hypothesis 1 implies that the rotations $T_{0,\varphi}$ act smoothly on the center-unstable bundle. Indeed, $T_{0,-\omega_* t}\, D\Phi_t(u_*, \mu_*)$ is a bounded group on the finite-dimensional space E_*^{cu}. On account of local compactness of E_*^{cu} and Hypothesis 1, this group is actually continuous in t. Thus, it is in fact analytic as it is given by e^{Bt} for some matrix B. We shall emphasize that the matrix B determines the linearization $D_v g_N(0, \mu_*)$ of the normal vector field g_N. Also, if $Y = C^0_{\text{eucl}}(\mathbb{R}^2, \mathbb{R}^n)$, we have $B = L|_{E_*^{cu}}$, see (5). Smoothness of the action of $SE(2)$ on the center bundle is now used to modify the vector field equivariantly and smoothly near the group orbit. Then an application of the graph transform shows the existence of the center-unstable manifold having the properties stated in Theorem 1. Finally, the representation of the flow on the manifold is obtained by considering the pull-back of the vector field to the covering space $SE(2) \times V_*$ of the center-unstable bundle. Here, the covering is defined by $(a, \varphi, v) \mapsto T_{a,\varphi}(u_* + v)$. Detailed proofs will appear elsewhere.

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