(1) Find a horseshoe in the cat map (draw the picture!).

(2) Compute the kneading invariant of an infinitely renormalizable unimodal map (see Ex. 3 §1.18, in [Devaney]).

(3) Show that the rotation number is a continuous function of a circle homeomorphism.

(4) Suppose that a homeomorphism \( f \) on a compact manifold is topologically transitive, that is, for all nonempty open sets \( U, V \) there exists an \( N \) such that \( f^N(U) \cap V \neq 0 \). Show that \( f \) has a dense orbit.

(5) For \( f : I \to I \) unimodal, \( I = [0, 1] \), assume that the critical point is periodic, \( f^\ell(c) = c \) for some \( \ell > 0 \). Show that \( f \) is not structurally stable, that is, for each \( \varepsilon > 0 \) there exists \( g \) with \( \sup |g - f| < \varepsilon \) such that we cannot find a homeomorphism \( h \) of \( I \) such that \( f \circ h = h \circ g \).

(6) Suppose \( f : I \to I \), continuous, possesses a periodic orbit of period \( p \geq 3 \), with the ordering

\[
 f(x_j) = x_{j+1}, x_j < x_{j+1}, j = 1, \ldots, p-1, f(x_p) = x_1.
\]

Conclude that \( f \) has periodic orbits of any period.

(7) Find the parameter-value \( \mu_* > 0 \) of the first period-doubling in the cubic family \( f(x) = \mu(x - x^3) \) and find an expansion for the period-two orbit \( x_1(\mu), x_2(\mu) \) for \( \mu \sim \mu_* \). Show how you compute the expansion using bifurcation theory: solve \( f(x) = y, f(y) = x \) near \( \mu = \mu_* \), \( x = y = x_* \), the fixed point at criticality using Lyapunov-Schmidt reduction. You may compare with a direct computation in Mathematica.

*4 points each exercise for extra credit, due on Wednesday, April 10, in class.*