Math 8502, Spring 2019

Dynamical Systems and Differential Equations II

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— *Homework 4* —

(1) Find a horseshoe in the cat map (draw the picture!).

(2) Compute the kneading invariant of an infinitely renormalizable unimodal map (see Ex. 3 §1.18, in [Devaney]).

(3) Show that the rotation number is a continuous function of a circle homeomorphism.

(4) Suppose that a homeomorphism f on a compact manifold is topologically transitive, that is, for all nonempty open sets U, V there exists an N such that $f^N(U) \cap V \neq 0$. Show that f has a dense orbit.

(5) For $f : I \to I$ unimodal, $I = [0, 1]$, assume that the critical point is periodic, $f^\ell(c) = c$ for some $\ell > 0$. Show that f is not structurally stable, that is, for each $\varepsilon > 0$ there exists g with $\sup |g - f| < \varepsilon$ such that we cannot find a homeomorphism h of I such that $f \circ h = h \circ g$.

(6) Suppose $f : I \to I$, continuous, possesses a periodic orbit of period $p \geq 3$, with the ordering

$$f(x_j) = x_{j+1}, x_j < x_{j+1}, j = 1, \ldots, p-1, f(x_p) = x_1.$$

Conclude that f has periodic orbits of any period.

(7) Find the parameter-value $\mu_* > 0$ of the first period-doubling in the cubic family $f(x) = \mu(x - x^3)$ and find an expansion for the period-two orbit $x_1(\mu), x_2(\mu)$ for $\mu \sim \mu_*$. Show how you compute the expansion using bifurcation theory: solve $f(x) = y, f(y) = x$ near $\mu = \mu_*$, $x = y = x_*$, the fixed point at criticality using Lyapunov-Schmidt reduction. You may compare with a direct computation in Mathematica.

4 points each exercise for extra credit, due on Wednesday, April 10, in class.