Math 8571/2, 3 Credits, Fall 2017/Spring 2018

Theory of Evolutionary Equations

— Syllabus —

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Time & Place MWF 11:15-12:05pm, VinH 213

Material Covered In this two-semester course, we develop tools to study partial differential equations from a dynamical systems point of view. We will learn how to "solve" time-dependent nonlinear partial differential equations using semigroup methods in order to study the qualitative behavior of solutions. We will use these methods to characterize stability of equilibria and periodic solutions, show existence of invariant manifolds, carry out regular and singular perturbation theory, and prove some bifurcation results. We will also address more global questions such as the existence of global attractors, ω-limit sets, and heteroclinic or homoclinic phenomena. We will then study phenomena that are genuinely associated with spatially extended dynamical systems, such as pattern formation, traveling fronts or pulses, and, more generally, existence, stability, and interaction of coherent structures. This second part will stress the importance of "pointwise methods", as opposed to the more abstract functional analytic methods in the first part. One objective is to develop the relations between pointwise bounds on the linearization at traveling waves, Evans function methods for pointwise Green's functions, and geometric properties of wave profiles.

The course focuses on parabolic, dissipative systems, with specific examples from the Ginzburg-Landau family, reaction diffusion systems, phase-field or Cahn-Hilliard systems, Swift-Hohenberg equations, and viscous conservation laws. We will also discuss lattice differential equations and interacting-particle systems as spatially discrete, extended dynamical systems. As time permits, I will mention some aspects of Hamiltonian or dispersive PDEs such as the Korteweg-de Vries equation or nonlinear Schrödinger systems.

Text No textbook is required. Many of the ideas for the first part of the course were first put together in Dan Henry's book "Geometric Theory Of Semilinear Parabolic Equations".

Office Hours MWF 9:00 – 10:00, VinH 509 or by appointment.

Contact All material regarding the course, in particular homework assignments, can be found on my homepage http://www.math.umn.edu/∼scheel. Everybody is encouraged to ask questions at any time, best during or after the lecture, at office hours, or by appointment. Best way to reach me is email to scheel@math.umn.edu.

Homework There will be 3 graded homework assignments.

Composition of Grade Average of homework assignments.