Quiz 1 MATH 1272

Any calculator that can display graphs, perform symbolic manipulations, or store text in memory is NOT allowed. Show all your work to receive full credit.

Problem 1  [10pt] Using integration by parts, find the definite integral \[
\int_0^\pi e^x \cos x \, dx
\]

Solution: First we look at indefinite integral \[
\int e^x \cos x \, dx
\]

Let \( u = \cos x \) and \( v = e^x \) we have \[
\int e^x \cos x \, dx = \int \cos x \, e^x.
\]

Then by integration by parts, we have
\[
\int e^x \cos x \, dx = e^x \cos x - \int e^x \sin x \, dx
\]

Applying integration by parts again, we have \[
\int e^x \sin x \, dx = \int \sin x \, e^x = e^x \sin x - \int e^x \cos x \, dx.
\]

Combining it with RHS of (1) we have:
\[
\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + C
\]

\[
\int_0^\pi e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) \bigg|_0^\pi = \frac{1}{2} e^\pi (-1 + 0) - \frac{1}{2} \cdot 1 \cdot (1 + 0) = -\frac{e^\pi + 1}{2}
\]

Problem 2  [10pt] Evaluate the integral \[
\int \sin^2 4x \cos^2 4x \, dx
\]

Solution: First try to simplify the integrand. \[
\sin^2 4x \cos^2 4x = (\sin 4x \cos 4x)^2 = \left(\frac{1}{2} \sin 8x\right)^2 = \frac{1}{4} \sin^2 8x = \frac{1}{4} \cdot \frac{1}{2} (1 - \cos 16x).
\]

Thus,
\[
\int \sin^2 4x \cos^2 4x \, dx = \frac{1}{8} \int 1 - \cos 16x \, dx = \frac{1}{8}x - \frac{1}{8} \int \cos 16x \, dx.
\]

Let \( u = 16x, \, dx = \frac{1}{16} \, du \), by substitution rule we have:
\[
\int \sin^2 4x \cos^2 4x \, dx = \frac{1}{8}x - \frac{1}{8} \frac{1}{16} \int \cos u \, du = \frac{1}{8}x - \frac{1}{128} \sin 16x + C.
\]