Math 1272-010 Calculus II
Midterm Exam III
April 23 2015

Name ______________

Discussion TA & Time ______________

Instructions:

• You have 50 minutes for seven problems.

• No books or notes are allowed.

• You may use a non-graphing calculator; no other electronic devices are permitted.

• Please show all your work on the exam paper. Little or no credit will be given for unsupported answers.

• Check your answers carefully.
**Test for Divergence** If $\lim_{n \to \infty} a_n$ does not exist or if $\lim_{n \to \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

**Integral Test** Suppose $f$ is continuous, positive, decreasing function on $[1, \infty)$ and $a_n = f(n)$. Then $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_{1}^{\infty} f(x) \, dx$ is convergent.

**Alternating Series Estimation** If $s = \sum (-1)^{n-1} b_n$ is the sum of an alternating series that satisfies
(i) $b_{n+1} \leq b_n$ and (ii) $\lim_{n \to \infty} b_n = 0$, then $|s - s_n| \leq b_{n+1}$.

**Comparison/Limit Comparison Test** Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.
(i) $\sum b_n$ conv. and $a_n \leq b_n$ for all $n$, then $\sum a_n$ is conv.
(ii) $\sum b_n$ div. and $a_n \geq b_n$ for all $n$, then $\sum a_n$ is div.
(iii) If $\lim_{n \to \infty} \frac{a_n}{b_n} = c$, where $0 < c < \infty$, then either both series conv. or div.

**Ratio Test** Suppose $\lim_{n \to \infty} |a_{n+1}/a_n| = L$.
(i) If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).
(ii) If $L > 1$ or $L \to \infty$, then $\sum_{n=1}^{\infty} a_n$ is divergent.
(iii) If $L = 1$, inconclusive.

**Root Test** Suppose $\lim_{n \to \infty} \sqrt[n]{a_n} = L$.
(i) If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).
(ii) If $L > 1$ or $L \to \infty$, then $\sum_{n=1}^{\infty} a_n$ is divergent.
(iii) If $L = 1$, inconclusive.
1. (20 pts) Use the Integral Test and Test for Divergence to show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$. You must evaluate the corresponding improper integral explicitly to get the full credits.
2. (15 pts) Determine whether the series is conditionally convergent, absolutely convergent or divergent.

\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}
\]

3. (15 pts) If the \( n \)th partial sum of a series \( \sum_{n=1}^{\infty} a_n \) is \( s_n = 3 - ne^{-n} \), find \( a_n \) and \( \sum_{n=1}^{\infty} a_n \).
4. (15 pts) Express $0.\overline{46} = 0.464646\ldots$ as a ratio of integers.
5. (15 pts)
(a) (10pts) Show that the series \( \sum_{n=1}^{\infty} (-1)^{n-1}ne^{-n} \) is convergent.
(b) (5pts) How many terms do we need to add in order to find the sum correct to one decimal places (\( |\text{error}| < 0.1 \))? (You don’t need to calculate the estimated sum).
6. (10 pts) Find the radius of convergence and interval of convergence of the series.

\[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]
7. (10 pts) Find a power series representation for the function and determine the radius of convergence.

\[ f(x) = \ln(2 - x) \]