Math 1272
Quiz 10
Thursday 30 April 2015

Scientific calculators are allowed but not required. Calculators with graphing and computer algebra capabilities are not allowed. Show your work and give exact answers.

1. (8 points) Find the Maclaurin expansion of

\[ f(x) = (1 - x)^{-2} \]

\[ f'(x) = (-2)(1-x)^{-3} \]
\[ f''(x) = (-6)(1-x)^{-4} \]
\[ f^n(x) = (n+1)! (1-x)^{-(n+2)} \]
\[ f^{(n)}(0) = (n+1)! \]

\[ f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \]

2. (8 points) Rewrite the following equation to show it is the equation of a sphere.

\[ 2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1 \]

\[ x^2 + y^2 + z^2 - 4x + 12z + \frac{1}{2} \]

\[ y^2 + (x^2 - 4x) + (z^2 + 12z) = \frac{1}{2} \]

\[ (x-2)^2 - 4 + y^2 + (z+6)^2 - 36 = \frac{1}{2} \]

\[ (x-2)^2 + y^2 + (z+6)^2 = 40 + \frac{1}{2} = \frac{81}{2} = \left(\frac{9}{\sqrt{2}}\right)^2 \]

\[ (x-2)^2 + y^2 + (z+6)^2 = \left(\frac{9}{\sqrt{2}}\right)^2 \]
3. (4 points) For the vectors \( \vec{a} = \langle 1, 2, 2 \rangle \) and \( \vec{b} = \langle 1, 1, 0 \rangle \)

Find the following:

(i) \( \vec{a} + \vec{b} = \langle 2, 3, 2 \rangle \) \( \text{1 pt} \)
(ii) \( \vec{a} - \vec{b} = \langle 0, 1, 2 \rangle \) \( \text{1 pt} \)
(iii) \( 2\vec{a} = \langle 2, 4, 4 \rangle \) \( \text{1 pt} \)
(iv) \( |\vec{a}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3 \) \( \text{1 pt} \)