1. (5 points) Set up an integral for the area of the surface obtained by rotating the curve \( y = e^x \), \( 0 \leq x \leq 1 \), about the \( x \)-axis.

\[
\begin{align*}
\frac{dy}{dx} &= e^x, \quad \left(\frac{dy}{dx}\right)^2 &= e^{2x} \\
S &= \int_0^1 2\pi e^x \sqrt{1 + e^{2x}} \, dx
\end{align*}
\]

2. (7 points) Show that every member of the family of functions

\[ y = ce^x \]

is a solution of the differential equation \( y' = y \). Here \( c \) is a constant.

\[
\begin{align*}
\frac{dy}{dx} &= ce^x, \\
y_1 &= ce^x \\
\text{So } y_1 &= ce^x \text{ is a solution to } y' = y.
\end{align*}
\]
3. (8 points) Find the center of mass of a semicircular plate of radius 1.

\[ y = \sqrt{1-x^2} \]

+2 Correct equation bounding semicircle

\[ A = \int_{-1}^{1} \sqrt{1-x^2} \, dx \]

\[ x = \sin \theta \]
\[ dx = \cos \theta \, d\theta \]
\[ l = \sin \theta \]
\[ \theta = \frac{\pi}{2} \]
\[ -l = \sin \theta \]
\[ \theta = -\frac{\pi}{2} \]

\[ A = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 \theta} \cos \theta \, d\theta \]

\[ = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta \]
\[ = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (1 + \cos 2\theta) \, d\theta \]
\[ = \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \bigg|_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \]
\[ = \frac{\pi}{4} + 0 - (\frac{\pi}{4} + 0) \]
\[ = \frac{\pi}{2} \]

or

\[ A \text{ is the area of a semicircle of radius 1} \]
\[ A = \frac{1}{2} \pi \cdot (1)^2 \]
\[ = \frac{\pi}{2} \] +2 \( A = \frac{\pi}{2} \)

+2 \( x = 0 \) by symmetry

\[ \bar{x} = \frac{1}{A} \int_{-1}^{1} x y(x) \, dx \]
\[ = \frac{2}{\pi} \int_{-1}^{1} x \sqrt{1-x^2} \, dx \]
\[ u = 1-x^2 \quad u(-1) = 0 \]
\[ du = -2xdx \quad u(0) = 0 \]
\[ = \frac{-1}{\pi} \int_{0}^{1} \sqrt{u} \, du \]
\[ = 0 \]

\[ \bar{x} = 0 \]

\[ \bar{y} = \frac{1}{A} \int_{-1}^{1} \left[ y(x) \right]^2 \, dx \]
\[ = \frac{2}{\pi} \int_{-1}^{1} (1-x^2) \, dx \]
\[ = \frac{\pi}{4} \left[ x - \frac{x^3}{3} \right]_{-1}^{1} \]
\[ = \frac{\pi}{4} \left( 1 - \frac{1}{3} - (1 - \frac{1}{3}) \right) \]
\[ = \frac{4}{3\pi} \]

+2 \( \bar{y} = \frac{4}{3\pi} \)

The center of mass is \((0, \frac{4}{3\pi})\).