Scientific calculators are allowed but not required. Calculators with graphing and computer algebra capabilities are not allowed. Show your work and give exact answers.

1. (6 points) Suppose the populations of rabbits and wolves are described by the Lotka-Volterra equations

\[
\frac{dR}{dt} = kR - aRW \quad \frac{dW}{dt} = -rW + bRW
\]

where \(k = 0.08\), \(a = 0.001\), \(r = 0.02\), and \(b = 0.00002\). Find all constant solutions to these equations.

\[
\frac{dR}{dt} = 0.08R - 0.001RW
\]

For constant solutions, \(\frac{dR}{dt} = \frac{dW}{dt} = 0\).

\[O = 0.08R - 0.001RW\]

\[= R(0.08 - 0.001W)\]

Either \(R = 0\) or

\[0.08 - 0.001W = 0\]

\[W = \frac{0.08}{0.001} = 80\]

This gives us two constant solutions: \(R = 0, W = 0\) and \(R = 1000, W = 80\).

2. (6 points)
   (a) Sketch the curve. Indicate with an arrow the direction in which the curve is traced as \(t\) increases.
   (b) Eliminate the parameter to find a Cartesian equation of the curve.

\[x = \sqrt{t}, \quad y = 1 - t\]

\[x = \sqrt{t}
\]

\[x^2 = t
\]

\[y = 1 - x^2
\]

\[x \geq 0\]

\[+2 \text{ Find } y = 1 - x^2
\]

\[+1 \text{ } x \geq 0
\]
3. (8 points) Solve the initial value problem:

\[ xy' = y + x^2 \sin x, \quad y(\pi) = 0 \]

\[
y' - \frac{1}{x} y = x \sin x \quad \text{[+1 Divide by } x]\]

\[
I(x) = e^{\int -\frac{1}{x} \, dx}
\]
\[
= e^{\ln x}
\]
\[
= e^{\ln x^1}
\]
\[
= x \quad \text{[+2 Find Integrating Factor]}
\]

Multiplying by our integrating factor, we get

\[
x^{-1}(y' - \frac{1}{x}y) = x^{-1}x \sin x
\]
\[
\frac{1}{x} y' - \frac{1}{x^2} y = \sin x
\]
\[
(\frac{1}{x} y)' = \sin x \quad \text{[+1 Use Product Rule]}
\]

\[
\int (\frac{1}{x} y)' \, dx = \int \sin x \, dx
\]
\[
\frac{1}{x} y = -\cos x + C \quad \text{[+2 Integrate]}
\]

Our initial condition gives us

\[
0 = y(\pi) = -\pi \cos \pi + C \pi
\]
\[
0 = \pi + C \pi
\]
\[
C = -1 \quad \text{[+2 Find } C]\]

So \[
y = -x \cos x - x
\]