

Math 8470, Topics in Mathematical Theory of Continuum Mechanics

MWF 01:25 PM - 02:15 PM, Vincent Hall 364

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Various geometric and variational structures in the equations of Classical Mechanics (such as the principle of least action, invariant formulations, canonical transformations, Hamilton-Jacobi theory, connections between symmetries and conservation laws, etc) were discovered by Euler, Lagrange, Hamilton, Noether and other classics. These structures have also played an important role in linking classical and quantum mechanics (in a suitable limit). More recently, ideas originating in this context were further developed by V.I. Arnold and his school, leading to important developments in symplectic geometry.

At a formal level many of the structures appear in PDEs arising in various classical field theories.

V. I. Arnold also pioneered the geometric approach to the study of incompressible fluids, in which the solutions or incompressible Euler equations are related to the geodesics in the group of volume-preserving diffeomorphisms. The development of the theory of completely integrable system in the 1960s and 1970s put emphasis on still another angle of these classical themes.

In this course we will explore some of these structures, one of the main motivating examples being V. I. Arnold's geometric formulation of the incompressible Euler equations. A good finite-dimensional analogue of it is represented by geodesics on Lie groups with right-invariant metrics, and can be used to illustrate a number of the classical concepts. An infinite-dimensional version of this situation not only leads to Euler's equations (for the groups of volume-preserving diffeomorphisms), but in simpler situations (such as the group of diffeomorphism of the circle) also leads to some well-known completely integrable equations in 1d.

The presentation will be self-contained as much as possible. Prerequisites essentially include only standard topics in multi-variable calculus. Elementary knowledge of differential forms and related notions might be useful, but is not necessary, and can be learned along the way.

There will be 3 homework assignments (and the grade will be based on the homework).

Please feel free to contact the instructor if you have any questions.