MATH 5587 (FALL2019): HOMEWORK 10

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Due Thursday, December 5

Problem 1.

Show that (assuming sufficient smoothness of the domain and the data) u is a solution to the Dirichlet boundary value problem

$$-\Delta u = f \text{ in } \Omega, \quad u|_{\partial\Omega} = g$$

if and only if u is a minimizer of the energy functional, that is,

$$E(u) = \min\{E(v) : v \in C^2(\bar{\Omega}) \text{ such that } v|_{\partial\Omega} = g\}.$$

Here,

$$E(u) = \int_\Omega \left(\frac{1}{2}|\nabla u|^2 - fu\right) dx.$$

The rest: 7.1.1f; 7.1.3a,b; 7.1.13; 7.1.20 a): (i), (iii), and b); 7.2.2a, 7.2.3d; 7.2.12; 7.3.4 from the usual textbook

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Date: December 1, 2019.