MATH 5587 (FALL2019): HOMEWORK 8

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Due Tuesday, November 26

Problem 1.

Find the eigenvalues and the eigenfunctions for the Dirichlet and Neumann problems for the Laplacian on a rectangle $(0, a) \times (0, b)$.

Problem 2.

Prove that the wave equation

$$\partial_t^2 u(t, x) = c^2 \Delta_x u(t, x), \quad t > 0, \quad x \in \Omega \in \mathbb{R}^d$$

with Dirichlet boundary conditions

$$u(t, x) = 0 \quad \text{for} \quad x \in \partial\Omega,\ t > 0,$$

has a solution

$$u(x, t) = \sum_n \Big(A_n \cos(\sqrt{\lambda_n} ct) + B_n \cos(\sqrt{\lambda_n} ct) \Big) v_n(x)$$

where λ_n and v_n are, respectively, eigenvalues and eigenfunctions of the Dirichlet problem for the Laplacian in Ω.

Write in an analogous form the solution to the heat equation

$$\partial_t^2 u(t, x) = c\Delta_x u(t, x), \quad t > 0, \quad x \in \Omega \in \mathbb{R}^d$$

with Dirichlet boundary conditions

$$u(t, x) = 0 \quad \text{for} \quad x \in \partial\Omega,\ t > 0.$$

The rest: 6.3.9, 6.3.10, 6.3.18, 6.3.21, 6.3.23, 6.3.27, 6.3.31 from the usual textbook

Date: November 21, 2019.