3.5.11e,f

(e) doesn't pass test; ★ (f) uniformly convergent.

3.5.21 a,c,e

(a) The periodic extension is not continuous, and so the best one could hope for is $a_k, b_k \to 0$ like $1/k$. Indeed, $a_0 = -2\pi,\ \ a_k = 0,\ \ b_k = (-1)^{k+1} 2/k,$ for $k > 0$.

(c) The periodic extension is C^0, and so we expect $a_k, b_k \to 0$ like $1/k^2$. Indeed,

$$a_0 = \tfrac{2}{3}\pi^2,\ \ a_k = (-1)^k 4/k^2,\ \ b_k = 0,\ \ \text{for } k > 0.$$

(e) The periodic extension is C^∞, and so we expect $a_k, b_k \to 0$ faster than any (negative) power of k. Indeed, $a_0 = 1,\ \ a_2 = -\frac{1}{2}$, and all other $a_k = b_k = 0$.

3.5.22 a,f

3.5.26 c,e

★ (c) converges in norm; ★ (e) does not converge in norm.

3.5.43

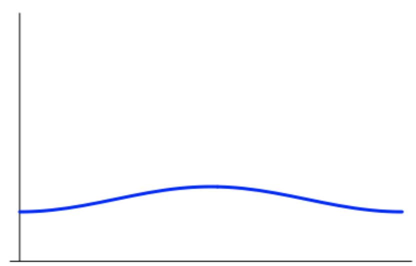
4.1.7

(a) $u(t,x) = \dfrac{1}{4} - \dfrac{2}{\pi^2} \displaystyle\sum_{j=0}^{\infty} \frac{1}{(2j+1)^2} \exp\left(-(4j+2)^2 \pi^2 t\right) \cos(4j+2)\pi x;$ (b) $\frac{1}{4}$;

(c) At an exponential rate of $e^{-4\pi^2 t}$;

(d) As $t \to \infty$, the solution becomes a vanishingly small cosine wave centered around $u = \frac{1}{4}$, namely

$$u(t,x) \approx \frac{1}{4} - \frac{2}{\pi^2} e^{-4\pi^2 t} \cos 2\pi x:$$



4.1.10 a, c

4.1.10. (a) $u(t,x) = e^{-t}\cos x$; equilibrium temperature: $u(t,x) \to 0$.

★ (c) $u(t,x) = \frac{1}{2}\pi - \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{e^{-(2k+1)^2 t}\cos(2k+1)x}{(2k+1)^2}$; equilibrium temperature: $u(t,x) \to \frac{1}{2}\pi$.

4.1.16 a, b

(a) If $u(t,x) = e^{\alpha t} v(t,x)$, then

$$\frac{\partial u}{\partial t} = \alpha e^{\alpha t} v(t,x) + e^{\alpha t}\frac{\partial v}{\partial t}(t,x) = \gamma e^{\alpha t}\frac{\partial^2 v}{\partial x^2} = \gamma \frac{\partial^2 u}{\partial x^2}.$$

(b) $v(t,x) = e^{-\alpha t}\sum_{n=1}^{\infty} b_n e^{-(\alpha+\gamma n^2\pi^2)t}\sin n\pi x$, where $b_n = 2\int_0^1 f(x)\sin n\pi x\,dx$ are the Fourier sine coefficients of the initial data. All solutions tend to the equilibrium value $u(t,x) \to 0$ as $t \to \infty$ at an exponential rate. For most initial data, i.e., those with $b_1 \neq 0$, the decay rate is e^{-at}, where $a = \alpha + \gamma\pi^2$; other solutions decay at a faster rate.