Math 8301, Manifolds and Topology
Homework 1
Due in-class on **Friday, Sep 12**

1. Prove that any open subset of a manifold is a manifold.

2. Prove that the product of an n-dimensional manifold with an m-dimensional manifold is an $(n+m)$-dimensional manifold.

3. For $A > a$, a torus of major radius A and minor radius a is defined by the equation
$$z^2 + \left[\sqrt{x^2+y^2} - A\right]^2 = a^2$$
in $\mathbb{R}^3$. Use cylindrical coordinates to show that this is homeomorphic to the quotient of the space $[0, 2\pi] \times [0, 2\pi]$ by the identifications $(\theta, 0) \sim (\theta, 2\pi)$ and $(0, \phi) \sim (2\pi, \phi)$. (Hint: Use the "continuous bijection from compact to Hausdorff" criterion as we did for S^1.)

4. (After Monday's lecture) For each value of $t \in \mathbb{R}$, decide whether the space
$$\{(x, y, z) \in \mathbb{R}^3 \mid xyz = t\}$$
is a manifold, and explain why or why not.

5. (After Monday's lecture) For which values of $t \in \mathbb{R}$ is the space
$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + xy + ty^2 = 1\}$$
a *compact* manifold?