Math 8301, Manifolds and Topology
Homework 2
Due in-class on **Friday, Sep 19**

1. Show graphically that the simplicial complex with 7 vertices, generated by the triangles below, gives rise to a space homeomorphic to the torus.

123	127	134	145	156	167	236
245	246	257	347	356	357	467

2. For a 2-dimensional simplicial complex with v vertices, e edges, and f triangles, the *Euler characteristic* χ is defined to be $v - e + f$. If this simplicial complex gives rise to a compact *surface*, give formulas for e and f which are nondecreasing in v in terms of χ and v.

3. Using the formulas from the previous problem, show that any triangulation of a compact surface of Euler characteristic 0 requires at least 7 vertices, and any of Euler characteristic 1 requires at least 6 vertices.

4. By identifying points on opposite sides of an icosahedron, give a simplicial complex triangulating $\mathbb{RP}^2$ (the surface obtained from S^2 by identifying (x, y, z) with $(-x, -y, -z)$) having 6 vertices and 10 faces. If you don't have easy access to an icosahedron for reference, the logo for the Mathematical Association of America is a picture of (the visible half of) one.

5. Suppose that you are given a description of a surface by edge identifications as in class: you have an ordered sequence of elements of the form a or a^{-1}, drawn from some set of letters A, such that each letter in A occurs exactly twice (ignoring the number of inverse signs that occur). Show that if a subsequence of the form $aabcb^{-1}c^{-1}$ occurs anywhere, you can get an equivalent description of the surface by replacing *just* this subsequence with $xxyyzz$ and leaving the rest alone (where x, y, z are new letters and a, b, c are thrown out). Either a description by cut-and-paste or by algebraic substitution are acceptable.