Math 8301, Manifolds and Topology
Homework 7
Due in-class on **Friday, Oct 31**

1. Show that S^2 is a universal covering space of $\mathbb{RP}^2$.

2. If $p : Y \to X$ is a covering space, generalize the action of the fundamental group $\pi_1(X, x)$ on $p^{-1}(x)$ to show that the assignment $x \mapsto p^{-1}(x)$ extends to a functor p^{-1} from the category $\Pi_1(X)$ to the category of sets.

3. Suppose $p : Y \to X$ and $p' : Y' \to X$ are covering maps, and $\phi : Y \to Y'$ is a homeomorphism such that $p'\phi = p$. Show that the functors p^{-1} and $(p')^{-1}$, from $\Pi_1(X)$ to the category of sets, are naturally isomorphic.

4. Suppose $p : Y \to X$ is a covering map and $f : Z \to X$ is an arbitrary continuous map. The *pullback* $Y \times_X Z \subset Y \times Z$ is the subspace

$$\{(y, z) \in Y \times Z \mid p(y) = f(z)\}.$$

   Show that the map $p' : Y \times_X Z \to Z$ sending (y, z) to z is a covering map.

5. There is no problem 5.