1. Show that the only possible nontrivial natural transformations $H^n(X;\mathbb{Z}) \to H^m(X;\mathbb{Q})$ occur when $m = nd$, and are of the form $\alpha \mapsto a\alpha^d$ for some $a \in \mathbb{Q}$.

2. Use rational cohomology to compute the rational homotopy groups $\pi_k(S^3 \vee S^3) \otimes \mathbb{Q}$ in dimensions $k = 1 \ldots 7$.

3. (This question is worth double.) A local coefficient system $\mathcal{A}$ on a space $X$ consists of the following data:

- A set of abelian groups $\{A_x\}$ for $x \in X$.
- For every path $\gamma$ starting at $x$ and ending at $y$, an isomorphism of abelian groups $\gamma_*: A_x \to A_y$ that only depends on the homotopy class of the path.

Given a local coefficient system $\mathcal{A}$ on $X$, define the singular chain complex with values in $\mathcal{A}$ by

$$C_n(X;\mathcal{A}) = \bigoplus_{\sigma: \Delta[n] \to X} A_{\sigma(1,\ldots,1)}.$$ 

Recall that the standard $n$-simplex $\Delta[n]$ is

$$\{(t_1,\ldots,t_n)|0 \leq t_1 \leq \cdots \leq t_n \leq 1\}.$$ 

Use the structure of a local coefficient system to define a boundary map $\partial: C_n(X,\mathcal{A}) \to C_{n-1}(X,\mathcal{A})$, and show that it satisfies $\partial \circ \partial = 0$. 

(The homology of the resulting chain complex is the homology of $X$ with coefficients in the coefficient system $\mathcal{A}$. In particular, if $E \to B$ is a fibration, the homology groups of the fibers form local coefficient systems on $B$, and there is a version of the Serre spectral sequence that works with no assumptions on $\pi_1(B)$ acting trivially on $H_*(F)$.)