1. Suppose $f \in C^p(X)$, $g \in C^q(X)$, and $x \in C_{p+q+r}(X)$. Show, using the definition of the cap product given in class, that

$$g \smile (f \smile x) = (f \smile g) \smile x \in C_r(X).$$

Show that this makes $C_*(X) = \bigoplus_n C_n(X)$ into a \textit{right} module over the ring $C^*(X) = \bigoplus_n C^n(X)$. (For this reason, Hatcher writes cap products with the terms reversed.)


3. Hatcher, exercise 22 on page 259.

4. Suppose that $M$ is a compact orientable manifold, and let $\Sigma M$ be its suspension

$$M \times [0, 1]/\{(x, 0) \sim (y, 0), (x, 1) \sim (y, 1)\}.$$

Show that $\Sigma M$ cannot be a manifold unless $M$ has the same homology as a sphere. (You may not assume that $M$ is connected.)