1. Suppose \( f : X \rightarrow Y \) is a map of 1-connected CW-complexes such that the induced map \( f_* : H_*(X) \rightarrow H_*(Y) \) is an isomorphism. Show that \( f \) is a homotopy equivalence. (Hint: Replace \( f \) with a cofibration and identify the first nonvanishing relative homotopy group.)

2. Suppose \( X = K(G, n) \) and \( Y = K(H, n) \) are based CW-complexes which are Eilenberg-Maclane spaces. Show that the functor \( \pi_n \) gives an isomorphism

\[
[X, Y]_* \rightarrow \text{Hom}(G, H).
\]

(Don’t assume that \( X \) and \( Y \) are necessarily constructed by the same procedure as in class.)

3. The topological group \( S^1 \) acts on the unit sphere \( S^{2n+1} \subset \mathbb{C}^{n+1} \) via

\[
\lambda \cdot (z_0, \cdots, z_n) = (\lambda z_0, \cdots, \lambda z_n)
\]

with quotient space \( \mathbb{C}P^n \). You may assume that this action has transverse slices. Compute \( \pi_k(\mathbb{C}P^n) \) in as large a range as you can.

Let \( \mathbb{C}P^\infty = \bigcup_n \mathbb{C}P^n \). What are the homotopy groups of \( \mathbb{C}P^\infty \)?

4. Same question as the previous problem with \( \{\pm 1\} \) acting on \( S^n \subset \mathbb{R}^{n+1} \) with quotient \( \mathbb{R}P^n \), and union \( \mathbb{R}P^\infty \).