Problem 1. Assume $\sum_{n=1}^{\infty} a_n$ is a series whose $k$th partial sum is $5 - \frac{k}{3^k}$. Find $a_k$. What is the sum of the series equal to?

Problem 2. Determine whether the series
$$\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{3 + 2^{-n}}$$
converges.

Problem 3. Show that the following series converges and find its sum:
$$\sum_{n=1}^{\infty} \frac{2^{n-1} + 3^n}{4^n}.$$

Problem 4. Determine whether the series
$$\sum_{n=1}^{\infty} 2ne^{-n}$$
converges.

Problem 5. Determine whether the series
$$\sum_{n=1}^{\infty} \frac{5n}{2n^2 - 5}$$
converges.

Problem 6. Does the series
$$\sum_{n=1}^{\infty} \left( \frac{n}{n + 1} \right)^n$$
converge?

Problem 7. Find the radius of convergence of the series
$$\sum_{n=1}^{\infty} \frac{8^n x^n}{(n + 3)^2}.$$

Problem 8. Find the radius of convergence of the series
$$\sum_{n=1}^{\infty} (-1)^n 3nx^n.$$
Problem 9. Evaluate the following indefinite integral as a power series and find the radius of convergence:
\[ \int \frac{x}{1+x^4} \, dx. \]

Problem 10. Starting from the formula \( \sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x} \), valid for \( |x| < 1 \), find the sum of the series
\[ \sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n}. \]

Problem 11. Find the Maclaurin series for \( x \cos x^3 \).

Problem 12. Find an equation of the sphere that passes through \((2, -1, 7)\) and has center \((1, -3, 5)\).

Problem 13. Find the unit vector in the direction of the vector \((-2, 3, -1)\).

Problem 14. For which values of \( x \) are the vectors \( \langle 3, 2, x \rangle \) and \( \langle 2x, 4, x \rangle \) are orthogonal?

Problem 15. Find the projection of the vector \((-2, 3, -1)\) onto a direction orthogonal to both of the vectors \( \langle 1, 0, -2 \rangle \) and \( \langle 0, 1, 1 \rangle \).