MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS
SAMPLE MIDTERM TEST I

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test.

Good luck!

Problem 1. Solve the initial value problem
\[ xy' + 5y = 7x^2, \quad y(2) = 5. \]

Problem 2. A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F. After 30 minutes the temperature of the cake is 140°F. When will it be 105°F? Assume Newton’s law of cooling holds.

Problem 3. A commercial fishery is estimated to have carrying capacity of 10 thousand pounds of certain kind of fish. Suppose the annual growth rate of the total fish population \( P \), measured in thousand pounds, is governed by the logistic equation
\[ \frac{dP}{dt} = \left(1 - \frac{P}{10}\right)P, \]
and initially there is a total of 2 thousand lbs of fish. What is the fish population after 1 year? (Use \( 10/(1 + 4 e^{-1}) \approx 4.048. \))

Problem 4. (1) Use Euler’s method with step size \( h = 1 \) to approximate the solution to the initial value problem
\[ \frac{dy}{dx} = x^2 + y^3, \quad y(0) = 1, \]
on the interval \([0, 1]\).
(2) Use the improved Euler method with same step size for the same problem.

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