Problem 1. Let $A$ be the $3 \times 3$ matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Find $A^{-1}$ and $A^{-1}A^T$ or show they do not exist. Use your answer to solve the following linear system:
\[
x_1 + x_3 = 1, \\
x_2 = 1, \\
x_1 + x_2 + 2x_3 = 1.
\]
Solving it in a different way will not receive partial credit.

Problem 2. Consider the following matrices:
\[
A = \begin{bmatrix} 4 & 2 & -13 \\ 2 & 1 & -7 \\ 3 & 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & -2 & 13 \\ 0 & -1 & 7 \\ 0 & 0 & -4 \end{bmatrix}.
\]
(1) Compute $\det A$. Is $A$ invertible? Explain your answer.
(2) Compute $\det(A + B)$. Is $A + B$ invertible? Explain your answer.

Problem 3. Let
\[
A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix}.
\]
(1) Find the reduced row-echelon form of $A$.
(2) What is the rank $\text{rank}(A)$ of $A$? What is the dimension of the null space $\text{Null}(A)$ of $A$?
(3) Find a basis of the column space $\text{Col}(A)$ of $A$.

Problem 4. Consider the initial value problem:
\[
y'' + 4y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = 2.
\]
Will $y(x)$ cross the $x$ axis at some $x \geq 0$? If yes, find all such $x$. 

Date: March 19, 2014; Corrected on March 26.