5. Let \( f(x) = 2x + x^4 \) for \( x \in [0, 5] \).
   (a) Write down the function \( G(x) \), which is the odd continuation for \( f(x) \). Specify what terms will be zero and non-zero in the Fourier expansion for \( G(x) \).

   (b) Write down the function \( V(x) \), which is the even continuation for \( f(x) \). Specify what terms will be zero and non-zero in the Fourier expansion for \( V(x) \).

6. Suppose \( f(x) \) is defined for \( x \in [0, 7] \), and \( f(x) = 2e^{-4x} \). Another function, \( F(x) \), is given by the following:

\[
F(x) = \sum_{n=0}^{\infty} a_n \cos(\pi nx / 7),
\]

where

\[
a_n = \frac{2}{\pi} \int_{0}^{7} 2e^{-4x} \cos\left(\frac{\pi nx}{7}\right) dx.
\]

What is the value of \( F(3) \)? What is the value of \( F(-2) \)?

7. Let us suppose that both ends of a string of length 25 cm are attached to fixed points at height 0. Initially, the string is at rest, and has the shape \( 4 \sin(2\pi x / 25) \), where \( x \) is the horizontal coordinate along the string, with zero at the left end. The speed of wave propagation along the string is 3 cm/sec. Write down the complete initial and boundary value problem for the shape of the string.

8. Let us suppose that the following boundary value problem is given:

\[
\frac{\partial^2 y}{\partial t^2} = 50 \frac{\partial^2 y}{\partial x^2}, \quad x \in [0, 100], \tag{1}
\]

\[
y(0, t) = y(100, t) = 0, \tag{2}
\]

\[
y(x, 0) = x^2(100 - x), \tag{3}
\]

\[
\frac{\partial y(x, 0)}{\partial t} = \begin{cases} x, & 0 \leq x \leq 25, \\ 1/3(100 - x), & 25 \leq x \leq 100. \end{cases} \tag{4}
\]

What is the speed of wave propagation along the string? What is the initial displacement of the string at point \( x = 20 \)? What is the initial velocity of the string at point \( x = 50 \)? At what point of the string is the initial velocity the largest?