4. (a) Setting \( \theta = 0 \) gives \((2/\pi) - (4/\pi) \sum_{m=1}^{\infty} 1/(4m^2 - 1) = 0 \) or \( \sum_{m=1}^{\infty} 1/(4m^2 - 1) = 1/2 \), a result also obtainable from the observation that \((4m^2 - 1)^{-1} = 1/2[(2m - 1)^{-1} - (2m + 1)^{-1}] \), so that the series telescopes. Setting \( \theta = \frac{1}{2} \pi \) gives 1 = \( (2/\pi) - (4/\pi) \sum_{m=1}^{\infty} (-1)^m/(4m^2 - 1) \), or \( \sum_{m=1}^{\infty} (-1)^m/(4m^2 - 1) = (\pi - 2)/4 \).

(b) Setting \( \theta = \pi \) gives \( \pi^2 = (\pi^2/3) + 4 \sum_{n=1}^{\infty} 1/n^2 \) or \( \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6 \); setting \( \theta = 0 \) gives 0 = \( (\pi^2/3) + 4 \sum_{n=1}^{\infty} (-1)^n/n^2 \) or \( \sum_{n=1}^{\infty} (-1)^n/n^2 = \pi^2/12 \).

(c) Setting \( \theta = 0 \) gives 1 = \( [(\sinh \pi b)/\pi] \sum_{n=-\infty}^{\infty} (-1)^n/(b - in) \). The \( n = 0 \) term is 1/b, and for \( n > 0 \) the sum of the \( n \)th and \((-n)\)th terms is 2b(1)/((b^2 + n^2)); thus 1 = \( [(\sinh \pi b)/\pi] \left[(1/b) + 2b \sum_{n=1}^{\infty} (-1)^n/(b^2 + n^2)\right] \), or \( \sum_{n=1}^{\infty} (-1)^n/(b^2 + n^2) = (\pi \text{csch} \pi b - 1)/2b^2 \). Setting \( \theta = \pi \) gives \( [(\sinh \pi b)/\pi] \sum_{n=-\infty}^{\infty} 1/(b - in) = \frac{1}{2}(e^{\pi b} + e^{-\pi b}) = \cosh \pi b \). (The function represented by the series is discontinuous at \( \theta = \pi \), so the sum of the series is the average of the left and right hand limits!) Again the \( n = 0 \) term is 1/b, and for \( n > 0 \) the sum of the \( n \)th and \((-n)\)th terms is 2b(1)/((b^2 + n^2)), so \( (1/b) + \sum_{n=1}^{\infty} 2b/(b^2 + n^2) = \pi \coth \pi b \) and hence \( \sum_{n=1}^{\infty} 1/(b^2 + n^2) = (\pi b \coth \pi b - 1)/2b^2 \).