**Problem 2.** Prove that if \( f \) is a continuous function from \( \mathbb{R}^n \) to \( \mathbb{R}^k \), then for any open subset \( U \) of \( \mathbb{R}^k \), the pre-image
\[
V = \{ x \in \mathbb{R}^n : f(x) \in U \}
\]
is an open set in \( \mathbb{R}^n \).

For every point \( x \in V \), take an open ball \( B(r, f(x)) \) which will be in \( U \). Since \( U \) is open, \( \exists \) a ball \( B(r, f(x)) \) contained in \( U \). Since \( f(x) \) is continuous at \( x \), for each \( \varepsilon > 0 \), in particular, \( \varepsilon = r \), \( \exists \) \( \delta > 0 \):
\[
| f(y) - f(x) | < \varepsilon = r \quad \text{whenever} \quad \| y - x \| < \delta,
\]

or, equivalently,
\[
f(y) \in B(r, f(x)) \quad \text{whenever} \quad y \in B(\delta, x).
\]
This shows that \( B(r, x) \subset V \) (because \( B(r, f(x)) \subset U \)).