Problem C. If \( p(x) \) is a polynomial of even degree with the leading coefficient \( a_{2n} > 0 \), then \( \lim_{|x| \to \infty} p(x) = \infty \). After that the argument is identical to the proof of Theorem 2.83a in the textbook.

Problem D. We will use sequential continuity, Theorem 1.15. If \( p \neq 0 \) or \( 1 \), then there will always be a sequence \( \{a_n\} \) of rationals converging to \( p \), for which the sequence \( \{f(a_n)\} \) will converge to \( 1 - p \), and a sequence \( \{b_n\} \) of irrationals converging to \( p \), for which the sequence \( \{f(b_n)\} \) will converge to \( 1 - p^2 \). Since for \( p \neq 0 \) or \( 1 \), we have \( 1 - p \neq 1 - p^2 \), this implies that \( f(x) \) will be discontinuous at such \( p \).

If \( p = 0 \) or \( 1 \), then the values of \( f(x) \) at both rationals and irrationals near \( p \) will be given by the two continuous functions, which both happen to converge to \( f(p) \). Thus, if we have any sequence \( \{c_n\} \) converging to \( p \), then the sequence \( \{f(c_n)\} \) will be made of two sequences, the rational subsequence and the irrational one, each of which converges to \( f(p) \). Therefore, the sequence \( \{f(c_n)\} \) will converge to \( f(p) \).

Problem F. (i) Given \( \varepsilon > 0 \), take \( \delta = \varepsilon / 2M \). Then for each \( x, y \), such that \( |x - y| < \delta \), we have \( |f(x) - f(y)| = |f'(c)(x - y)| \leq M\delta = \varepsilon / 2 < \varepsilon \).

(ii) Take \( \varepsilon = 1 \). Then for any \( \delta > 0 \), take a natural number \( n \) such that \( n(2n + 1) > 1/\delta \), so that \( x = 1/n \) and \( y = 1/(n + 1/2) \) will be less than \( \delta \) apart. Then \( |\sin \pi / x - \sin \pi / y| = |0 - \sin \pi (n + 1/2)| = 1 \geq \varepsilon \). This implies the function is not uniformly continuous.