MATH 5335: GEOMETRY I
SAMPLE MIDTERM TEST I

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. For this sample test, pretend you are in a test situation and time yourself for 100 minutes, which is how long the actual test will be.

Good luck!

Problem 1. Find the point of intersection of the lines

$$\{t(5,4)\colon t \in \mathbf{R}\} \qquad \text{and} \qquad \{X\colon \langle (3,-1), X\rangle = -4\}\,.$$

Express the coordinates of your answer as fractions.

Problem 2. Decide if the line segment with endpoints $(17,12)$ and $(11,9)$ meets the line $\{t(7,5)\colon t \in \mathbf{R}\}$ and, if so where.

Problem 3. Let l denote the line with normal form

$$\langle (-3,4)\,,\, X\rangle = -3\,.$$

Let k denote the line through $(5,4)$ perpendicular to l. Find a point on k different from $(5,4)$ whose distance from l is the same as the distance of $(5,4)$ from l.

Problem 4. Let p and r be rays emanating from the origin with direction indicators $U=(1,0)$ and $W=(-8,15)$. Find a *unit* direction indicator V for the ray q which bisects $\angle(p,r)$.

Problem 5. Construct the matrix formula for an isometry which sends $\angle(5,2)(2,2)(4,4)$ to $\angle(-1,4)(-1,1)(-3,3)$.

Answer:

$$\begin{aligned}[\mathcal{U}(X)] &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left([X] - \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} [X] + \begin{bmatrix} 1 \\ -1 \end{bmatrix}.\end{aligned}$$

Problem 6. When developing the theory of isometries, we proved that for any isometry $\mathcal{U}$, any points P and Q, and any real numbers a and b for which $a+b=1$, the following relation holds:

$$\mathcal{U}(aP+bQ) = a\,\mathcal{U}(P) + b\,\mathcal{U}(Q)\,.$$

Prove that, as a consequence of this relation that, for any isometry, the image of any line is a line. [If you use a fact from Chapter 1, state it; you are NOT asked to prove that fact or give a reference for it.]

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Problem 7. Construct the matrix formula for the reflection across the line

$$l = \{(4, 2) + t(3, 5) \ : \ t \in \mathbb{R}\}.$$

Problem 8. You are being told that the following matrix formula represents a *glide reflection*, that is to say, a composition (in either order) of the reflection in a certain line l and the translation by a certain vector P parallel to that line:

$$[X] \rightsquigarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} [X] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} .$$

Find both the vector P and the mirror l.

Answer: $P = (1/2, 1/2)$ and $l = \{(1/4, -1/4) + t(1, 1) \ : \ t \in \mathbb{R}\}$.