You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. For this sample test, pretend you are in a test situation and time yourself for 100 minutes, which is how long the actual test will be.

Good luck!

**Problem 1.** Let \( A = (6, 0) \), \( B = (0, -6) \), \( C = (0, 0) \), \( X = (0, -3) \), \( Y = (4, 0) \). Find \( Z \) on \( AB \) such that \( AX \), \( BY \), and \( CZ \) meet at a common point.

**Problem 2.** Fill in each blank in (a), (b), (c) and (d) with a single word; no work need be shown.

(a) The medians of any triangle meet at the ____________ of that triangle.

(b) The perpendicular bisectors of the sides of a triangle meet at the ____________ of that triangle.

(c) The following three points are collinear: the centroid, the orthocenter, and the ____________ of any triangle.

(d) Two types of isometries whose fixed-point sets are empty are glide reflections and ____________.

**Problem 3.** For any isosceles triangle whose largest angle has measure \( 3\pi/4 \) prove that the ratio of the length of the longest side to that of the shorter sides is \( \sqrt{2 + \sqrt{2}} \).

**Hint:** The simplest approach involves the cosine function but no trigonometric identities.

**Problem 4.** For an arbitrary triangle \( \triangle ABC \) find the barycentric coordinates (in terms of the side lengths) of the point where the angle bisector of the angle at \( C \) meets the side \( \overrightarrow{AB} \). [You may use the fact that the barycentric representation of the incenter is

\[
\left( \frac{a}{a+b+c}, \frac{b}{a+b+c}, \frac{c}{a+b+c} \right)
\]

where \( a, b, \) and \( c \) are the lengths of the sides opposite \( A, B, \) and \( C, \) respectively.]

**Problem 5.** Give an example of a convex quadrangle \( ABCD \) that is not a parallelogram but which has the properties that \( \overrightarrow{AB} \) is parallel to \( \overrightarrow{CD} \) and \( |\overrightarrow{BC}| = |\overrightarrow{DA}| \). A clear picture with appropriate labels will suffice (even if not drawn very well).

**Problem 6.** Let \( P = (5, 5) \). Find a point \( Q \) on the circle \( \|X\| = 1 \) such that the line \( \overrightarrow{PQ} \) is tangent to the circle.

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Problem 7. Under inversion in the circle of radius 2 centered at (0, 0), where does the point (3, 0) get mapped? That is, find $\mathcal{I}(3, 0)$.

Problem 8. Find the equation of the Poincaré line that is incident with the points (0, 2) and (3, 7).