MATH 5335: GEOMETRY I
SAMPLE MIDTERM TEST II

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You may not use a calculator, notes, books, etc. Only the exam paper and a pencil or pen may be kept on your desk during the test. For this sample test, pretend you are in a test situation and time yourself for 100 minutes, which is how long the actual test will be.

Good luck!

Problem 1. Let $A = (6, 0)$, $B = (0, -6)$, $C = (0, 0)$, $X = (0, -3)$, $Y = (4, 0)$. Find Z on $\overline{AB}$ such that $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$ meet at a common point.

Problem 2. Fill in each blank in (a), (b), (c) and (d) with a single word; no work need be shown.

(a) The medians of any triangle meet at the ____________________ of that triangle.

(b) The perpendicular bisectors of the sides of a triangle meet at the ____________________ of that triangle.

(c) The following three points are collinear: the centroid, the orthocenter, and the ____________________ of any triangle.

(d) Two types of isometries whose fixed-point sets are empty are glide reflections and ____________________.

Problem 3. For any <u>isosceles</u> triangle whose largest angle has measure $3\pi/4$ prove that the ratio of the length of the longest side to that of the shorter sides is $\sqrt{2+\sqrt{2}}$. *Hint:* The simplest approach involves the cosine function but no trigonometric identities.

Problem 4. For an arbitrary triangle $\triangle ABC$ find the barycentric coordinates (in terms of the side lengths) of the point where the angle bisector of the angle at C meets the side $\overline{AB}$. [You may use the fact that the barycentric representation of the incenter is

$$\left(\frac{a}{a+b+c}, \frac{b}{a+b+c}, \frac{c}{a+b+c}\right)^{\triangle},$$

where a, b, and c are the lengths of the sides opposite A, B, and C, respectively.]

Problem 5. Give an example of a convex quadrangle $ABCD$ that is not a parallelogram but which has the properties that $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$ and $|\overline{BC}| = |\overline{DA}|$. A clear picture with appropriate labels will suffice (even if not drawn very well).

Problem 6. Let $P = (5, 5)$. Find a point Q on the circle $\|X\| = 1$ such that the line $\overleftrightarrow{PQ}$ is tangent to the circle.

Date: November 9, 2011.

Problem 7. Under inversion in the circle of radius 2 centered at $(0, 0)$, where does the point $(3, 0)$ get mapped? That is, find $\mathcal{I}(3, 0)$.

Problem 8. Find the equation of the Poincaré line that is incident with the points $(0, 2)$ and $(3, 7)$.