$$a_n = \begin{cases} 1, & \text{if } n \text{ odd} \\ n & \text{if } n \text{ even} \end{cases}$$

Not bounded, but

$\lim a_n \neq \infty$

Actually $\lim a_n$ does not exist,

$$\lim_{n\to\infty} \frac{a_n}{1+a_n} = \lim \frac{1}{1+\frac{1}{a_n}}$$

$\lim_{n\to\infty} 1/a_n = 0$

$\sum \frac{a_n}{1+a_n}$ conv.

$\Rightarrow \lim_{n\to\infty} \frac{a_n}{1+a_n} = 0$

$$\Rightarrow \lim_{n\to\infty} a_n = ?$$

$$\frac{a_n}{1+a_n} > \frac{a_n}{2}$$

$$\lim_{n\to\infty} \frac{a_n}{1+a_n} = 0 \Rightarrow \lim_{n\to\infty} a_n = 0$$

$\lim b_n = 0$, where

$$\boxed{b_n := \frac{a_n}{1+a_n}}$$

$$(1+a_n) b_n = a_n$$

$$a_n (b_n - 1) = -b_n$$

$$a_n = \frac{b_n}{1-b_n}$$

$$\lim a_n = \lim \frac{b_n}{1-b_n} = 0$$

$0 \le a_n \le M$

$\exists M: \forall n > M$

$$\frac{a_n}{1+a_n} > \frac{1}{1+M} \cdot a_n \quad \text{for } n > M$$