Math 5615 Honors: Problem-Solving Session

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Plan for Today

I would like you to split into five-person breakout rooms. Before splitting into breakout rooms, I will share a list problems with you via a pdf file on chat. Within your breakout room, you will do some brainstorming and group work and create solutions.

You will be getting your class point just for participating in the breakout rooms for the whole duration of class.

I will be visiting the breakout rooms to answer questions, give you hints, and accept your solutions.

Working in a Breakout Room

In your breakout room, do brainstorming altogether to get an idea of the solution. When creating the solution, try to see if it stands critique, try to be a devil's advocate. Choose the **Scribe**, who will write the solution up and the **Speaker**, who will present the report to me.

Problem 1

Suppose that $a_n > 0$, and $\sum_{n=1}^{\infty} a_n$ diverges. Prove that the following series must also diverge:

$$\sum_{n=1}^{\infty} \frac{a_n}{1 + a_n}.$$

Problem 2

Show that for any sequence $\{a_n\}$ of real numbers,

$$\liminf_{n\to\infty} a_n \leq \limsup_{n\to\infty} a_n.$$

Problem 3

Is there a metric space which is countable and compact?

Problem 4

Is the function $x \sin x$ uniformly continuous on the real line? Explain your answer.