Math 5615 Honors: Problem-Solving Session

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I would like you to split into two-three-person breakout rooms. Before splitting into breakout rooms, I will share a list problems with you via a pdf file on chat. Within your breakout room, you will do some brainstorming and group work and create solutions.

You will be getting your class point just for participating in the breakout rooms for the whole duration of class.

I will be visiting the breakout rooms to answer questions, give you hints, and accept your solutions.

Working in a Breakout Room

In your breakout room, you will play rotating roles:

The Host
: One person leads the talking within the group. Not that the other should shut up: that person will play the role of host, organizer, and mediator.

The Scribe
: One person writes it up: on a piece of paper or in a file.

The Speaker
: One person presents the report to me, when the report is ready.

Problem 1

Suppose you take a map of Minneapolis on a square piece of paper, take a smaller, 1/10 scaled photocopy of it and throw it on the original map so that the the smaller map lies entirely inside the larger. Prove that you can take a pin and puncture through a location on the smaller map which will be the same location on the larger map. How many locations like that do you expect to find?

Assume that the function $f : (0, \infty) \to \mathbb{R}$ is continuous. Assume also that $\lim_{x \to 0+} f(x)$ and $\lim_{x \to \infty} f(x)$ exist and are finite. Prove that f is bounded on $(0, \infty)$.

Problem 3

Suppose that $a_n > 0$, and $\sum_{n=1}^{\infty} a_n$ diverges. Prove that the following series must also diverge:

$$\sum_{n=1}^{\infty} \frac{a_n}{1 + a_n}.$$

Show that for any sequence $\{a_n\}$ of real numbers,

$$\liminf_{n\to\infty} a_n \leq \limsup_{n\to\infty} a_n.$$