Math 5615H **Homework 1**

Posted: 09/11; due Friday, 09/18

The problem set is due at the beginning of the class on Friday. Please scan handwritten pages and upload the resulting image file or the pdf file produced by LaTeX of whatever document preparation software you use to Canvas.

Reading: The syllabus. **Class notes.** **(Baby) Rudin**: Sections 2.1-2.6, 2.8-13, Chapter 1 through Section 1.6, 1.12-1.16, the *Schröder-Bernstein theorem* article in Wikipedia.

Problems:

1. (The Russell paradox) A *paradox* is a logically contradictory statement. This problem shows that one cannot say just anything in order to define a set: some care is required. Let us say that a set is *respectable* if it does not contain itself as an element. Let B be the set of all respectable sets. Try to see if B is respectable. The attempt to answer this leads to a paradox:

(1) Show that if B is respectable, then it must not be respectable;
(2) Show that if B is not respectable, then it must be respectable.

2. (No division by zero) Show that in a field, the additive identity 0 has no multiplicative inverse.

3. Show that there could be no bijection between a finite set and a proper subset of it. *Hint*: Use the pigeonhole principle.

4. Show that the set of polynomials with integral coefficients is countable.

5. Prove that if A is an infinite set, then A has a countable subset.

6. Show: if A is a set (including the empty set), then there is no bijection between A and the set $P(A)$ of all subsets of A. (This implies that $|P(A)| \neq |A|$ and, in fact, $|P(A)| > |A|$, and you get an infinite sequence of infinite cardinalities.) *Hint*: Assume there is a bijection $f : A \to P(A)$ and then construct a subset of A which could not be $f(a)$ for any $a \in A$.

7. Suppose a set A is infinite and B is countable. Show that $A \cup B \sim A$. *Hint*: Start with selecting a countable subset in A.