Math 5615H **Homework 5**

Posted: 11:30 p.m., 10/10, typo corrected: 10/12; due: Friday, 10/17/2014

The problem set is due at the beginning of the class on Friday.

Reading: Chapter 3: pages 47-55.

Problem 1. Let A be a subset of $\mathbb{R}$ such that $\inf A = 3$ and let $B = \{x^2 \mid x \in A\}$, the set of squares of numbers from A. Show that $\inf B = 9$.

Problem 2. Suppose x and y are two points in a metric space, such that $d(x, y) < \frac{1}{n}$ for any natural n. Show that $x = y$.

Problem 3. Suppose a set A is infinite, B is finite, and $A \cap B = \emptyset$. Show that the following cardinalities are equal: $|A \cup B| = |A|$, *i.e.*, show that $A \cup B$ is equivalent to A.

Problem 4. Show that the set of limit points of a subset of a metric space is closed.

Problem 5. Show that $[0, 1] \setminus \{\text{Cantor set}\}$ is dense in $[0, 1]$.

Problem 6. If A is a connected set in a metric space, show that its closure $\bar{A}$ is also connected.

Problem 7. Let $\{p_n\}$ be a Cauchy sequence in a *discrete* metric space X. Figure out what it might mean that such a sequence is *eventually constant* and show that it actually is.

Problem 8. Is any discrete metric space complete? Explain your answer.

Problem 9. Suppose that $\{p_n\}$ is a sequence of real numbers with limit p and $a \leq p_n \leq b$ for all n. Prove that $a \leq p \leq b$.