MATH 5616H: INTRODUCTION TO ANALYSIS II
SAMPLE MIDTERM EXAM I

You may not use notes, books, etc. Only the exam paper, a pencil or pen may be kept on your desk during the test. Calculators are not allowed, either, but will not be needed. Ask me, and I will compute anything for you, if you need me to. Unless stated otherwise, please show all of your work and justify your answers in order to receive full credit.

Good luck!

Problem 1. Suppose $f(x)$ is a continuous, real-valued function on an interval $[a, b]$. Show that there is a point y on this interval so that

$$f(y) = \frac{1}{b-a} \int_a^b f(x)dx.$$

Problem 2. Assume that $f_n \to f$ uniformly on $\mathbb{R}$ for a sequence $\{f_n\}$ of functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose that, for some $x_0 \in \mathbb{R}$ and each $n \in \mathbb{N}$, there exists a limit $a_n := \lim_{x \to x_0} f_n(x)$. Prove that the limit $\lim_{n \to \infty} a_n$ exists. (No need to find it.)

Problem 3. For $x \in [0, 1]$ define f_n by the formula

$$f_n(x) = \frac{3nx}{1 + n^2x^2}.$$

Prove that the family $\{f_n\}$ is not equicontinuous.

Problem 4. If the radius of convergence of a power series is R, $0 < R < \infty$, is it true that the series converges uniformly on $(-R, R)$? If yes, prove it. If not, give a counterexample and explain why it works.

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