Math 8202

Date due: March 1, 2010

Hand in only the starred questions. One of the other problems may show up on the quiz on March 8, the International Women’s Day. :-)

If you want me to do in class any of the questions I have given you, do let me know (on or after the due date for the assignment).

Section 5.1: 1, 2, 4*, 5, 6, 18.

Section 5.4: 2, 4, 7*, 10, 11, 13, 15, 17, 19.

N*. Show that every group of order 1001 is cyclic.

O. Let \( G \) be the group of all isometries of the cube, and let \( H \) be the subgroup consisting of rotations which preserve the cube. Let \(-1\) denote the element of \( G \) which is the transformation of \( \mathbb{R}^3 \) given by multiplication by \(-1\).

1. Show that \( G = H \times (-1) \). [For subgroups \( H \) and \( K \) of a group \( G \), we write \( G = H \times K \) to emphasize that \( G \) is the internal direct product of \( H \) and \( K \), i.e., \( G = HK \), the subgroups are normal and intersect trivially.]

2. Show that if \( g \in G \) is any element of order 2 other than \(-1\), then \( G \neq H\langle -1 \rangle \). [To do this you may need to prove that the center of \( H \) is \{1\}. Either use the isomorphism with \( S_4 \) or note that if you conjugate one rotation by another rotation you get rotation about an axis obtained by applying the second rotation to the axis of the first.]

P*. 

1. Let \( G \) be the group of all isometries of the tetrahedron, and let \( H \) be the subgroup consisting of rotations. Determine whether or not \( G = H \times K \) for some subgroup \( K \) of \( G \).

2. Let \( G \) be the group of all isometries of the icosahedron, and let \( H \) be the subgroup consisting of rotations. Determine whether or not \( G = H \times K \) for some subgroup \( K \) of \( G \).

Q. Show that the group \( \text{Aff}(V) = \{ x \mapsto Ax + b \mid x \in V, A \in \text{GL}(V), b \in V \} \) of affine transformations of a vector space \( V \) is a semidirect product \( V \rtimes \text{GL}(V) \).

R*. Show that the group \( Z_4 \) gives an example of an extension \( Z_2 \trianglelefteq Z_4 \) which is not isomorphic to a semidirect product \( Z_2 \rtimes Z_2 \).