Math 8212 **Homework 2**
Posted: 2/4/2020; Comments: 2/11 and 2/13; Due: Friday, 2/14/2020

The problem set is due at the beginning of the class on Friday, February 14.

All homework is assigned based on Eisenbud's third printing (1999 or later). If you are using an older printing, make sure to check the correction lists on our class home page against the assigned exercises.

Reading: **Class notes**. Remember that our approach to spectral sequences was dramatically different from that of Eisenbud. Thus, your class notes will be your best friend. **Eisenbud**: Sections A3.13.1, A3.13.3 up to Theorem A3.22, A3.13.4 through *i. Balanced Tor*, 8.1, and 9.0.

Problem 1. In dealing with the spectral sequence of a double complex $(C_{\bullet,\bullet}, \partial_h, \partial_v)$, show that if for $b \in C_{p,q}$ which has survived to E^2, we have $d^2[b]^2 = 0$, then there exist c_1 and c_2 so that b can be extended to a zigzag, such as the one we sketched in class:

$$\begin{aligned} \partial_v b &= 0, \\ \partial_h b &= -\partial_v c_1, \\ \partial_h c_1 &= -\partial_v c_2. \end{aligned}$$

Problem 2. Eisenbud's Exercise A3.36. Assume here that (F, d) is a differential graded (dg-) module (*i.e.*, a chain complex of modules) and the filtration is increasing:

$$0 = F_0 \subset F_1 \subset F_2 \subset F_3 = F.$$

Use the homological spectral sequence. Remember we defined, for each pair (p, q), the module $E^\infty_{p,q}$ to be just $E^r_{p,q}$, if it happens that the latter becomes independent of r for large enough r. (If you do this problem for a cochain complex with a decreasing filtration and run the cohomological spectral sequence, it will be fine, too.)

Problem 3. Eisenbud's Exercise A3.37. Here Eisenbud means the spectral sequence of the double complex obtained from the short exact sequence of complexes:

$$0 \to F \to F \to F/pF \to 0,$$

where the first map is multiplication by p.

Problem 4. Eisenbud's Exercise A3.41. The exercise would not hold the way it is worded without certain assumptions on the filtrations. Since we stated convergence of a spectral sequence for a bounded filtration (F_pC_n is bounded in p for each n, starting at 0 and ending at C_n), assume this for this problem. Use increasing filtrations, if you choose to deal with chain complexes, and use decreasing filtrations, if your complexes are cochain ones.

Problem 5. Eisenbud's Exercise 9.1.