# MATH 8254: ALGEBRAIC GEOMETRY
# PROBLEM SET 1, DUE FRIDAY, FEBRUARY 8, 2008

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**From Hartshorne's textbook:** II.1.22, II.2.14 (b) (finish what we have not done in class, i.e., construct a morphism of schemes), II.5.7 (c), II.5.9 (c), II.5.14 (a,b,c) (skip the $d$-uple embedding, hint: see the second half of the proof of Theorem 5.19).

**Problem 1.** For a graded ring $S = \bigoplus_{n=0}^{\infty} S_n$, show that the natural morphism $\operatorname{Proj} S \to \operatorname{Spec} R$, where $R = S_0$, is separated. [Hint: Show that for each homogeneous $f, g \in S_+$, there is a closed immersion $D_+(fg) \to D_+(f) \times_{\operatorname{Spec} R} D_+(g)$. This is enough, because the fibered products of the basic open sets $D_+$ provide a base of topology of $X \times_{\operatorname{Spec} R} X$.]

**Problem 2.**

(1) Show that a sheaf of $\mathcal{O}_X$-modules over a scheme $X$ is quasi-coherent, if and only if it is locally *presentable*, i.e., for each point $x$ in $X$, there is an open neighborhood $U$ of $x$ and sets $I$ and $J$, so that there is an exact sequence of $\mathcal{O}_U$-modules:

$$\mathcal{O}_U^I \to \mathcal{O}_U^J \to \mathcal{F}|_U \to 0.$$

(2) Show that a sheaf of $\mathcal{O}_X$-modules over a noetherian scheme $X$ is coherent, iff it is locally *finitely presentable*, which is the same as above with the sets $I$ and $J$ being finite.

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