MATH 8254: ALGEBRAIC GEOMETRY
PROBLEM SET 1, DUE FRIDAY, FEBRUARY 8, 2008

SASHA VORONOV

From Hartshorne’s textbook: II.1.22, II.2.14 (b) (finish what we have not
done in class, i.e., construct a morphism of schemes), II.5.7 (c), II.5.9 (c), II.5.14
(a,b,c) (skip the d-uple embedding, hint: see the second half of the proof of Theorem
5.19).

Problem 1. For a graded ring \( S = \bigoplus_{n=0}^{\infty} S_n \), show that the natural morphism
\( \text{Proj} S \to \text{Spec} R \), where \( R = S_0 \), is separated. [Hint: Show that for each homoge-
neous \( f, g \in S_+ \), there is a closed immersion \( D_+(fg) \to D_+(f) \times_{\text{Spec} R} D_+(g) \). This
is enough, because the fibered products of the basic open sets \( D_+ \) provide a base
of topology of \( X \times_{\text{Spec} R} X \).]

Problem 2. (1) Show that a sheaf of \( \mathcal{O}_X \)-modules over a scheme \( X \) is quasi-
coherent, if and only if it is locally \emph{presentable}, i.e., for each point \( x \) in \( X \),
there is an open neighborhood \( U \) of \( x \) and sets \( I \) and \( J \), so that there is an
exact sequence of \( \mathcal{O}_U \)-modules:
\[
\mathcal{O}_I \to \mathcal{O}_J \to \mathcal{F}|_U \to 0.
\]

(2) Show that a sheaf of \( \mathcal{O}_X \)-modules over a noetherian scheme \( X \) is coherent,
iff it is locally \emph{finitely presentable}, which is the same as above with the sets
\( I \) and \( J \) being finite.

Date: February 4, 2008.