MATH 8254: ALGEBRAIC GEOMETRY
PROBLEM SET 2, DUE MONDAY, MARCH 3, 2008

SASHA VORONOV

From Hartshorne’s textbook: II.5.11, II.7.8.

Problem 1. (1) Let $S = \bigoplus_{n=0}^{\infty} S_n$ be a graded ring and $R = S_0$. Suppose $\phi: R \to A$ is a ring homomorphism and let $T = S \otimes_R A$ be the graded ring induced by the base change. Prove that $\text{Proj } T \cong \text{Proj } S \times \text{Spec } R \times \text{Spec } A$.

(2) Let $S = \bigoplus_{n=0}^{\infty} S_n$ be a quasi-coherent graded $\mathcal{O}_X$-algebra over a scheme $X$. Let $f^*S = \bigoplus_{n=0}^{\infty} f^*S_n$ be the inverse image under a scheme morphism $f: Y \to X$. Prove that $\text{Proj } f^*S \cong \text{Proj } S \times_X Y$.

Problem 2. For a quasi-coherent sheaf $\mathcal{E}$ and an invertible sheaf $\mathcal{L}$ over a scheme $X$, prove that $\mathbb{P}(\mathcal{E})$ and $\mathbb{P}(\mathcal{E} \otimes \mathcal{L})$ are isomorphic schemes over $X$.

Problem 3. Let $f: Y \to X$ be a relative scheme. For an $f$-very ample sheaf $\mathcal{L}$ on $Y$, prove that for each integer $n > 0$, the sheaf $\mathcal{L}^{\otimes n}$ is also $f$-very ample.

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